CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2026

Contents

9:10 AM Classes

******* ******* (🌶️🌶️, August, TWΘFS)
Well you have to just come to find out the title of the course. However, here's a glimpse into the content of the course. You will be solving these problems:

(1) Decompose the real numbers into 163 disjoint sets, each closed under addition.
(2) Can you write $f(x) = x^2$ into the sum of 3 periodic functions?
(3) If a rectangle is tiled by square tiles, show that its sides have a rational ratio.
(4) Even a Hilbert problem!

(All these at 2 chili, can you believe?)

The theme/through line/common techniques amongst these problems is the title of this course. You just have to be there. Is there any reason not to come? I think not. But I'm biased. Ask others.

Homework: Recommended

Class format: Lecture for the first class, then casual problem-solving based and very interactive.

Prerequisites: Knowing the definitions of vector spaces, linear maps and basis would be very helpful. There will be a quick recap of these concepts on the first day. -1 chili on the first day if you have seen these before.

Calendars and Balanced Words (🌶️, Nikita, TWΘFS)
The dates of some holidays, including Lunar New Year and Easter, are calculated according to rules involving several cycles: the lunar cycle, the solar cycle, and the seven-day week. These calculations were once a way for mathematicians to make a living. We will discuss some of the mathematics related to rounding real arithmetic progressions. Similar patterns also occur in screensavers and floating-point operations.

Homework: Recommended

Class format: Lecture + a bit of problem solving

Prerequisites: None.

Can You Draw a Graph Nicely? (🌶️🌶️🌶️, Bowen, TWΘFS)
You may have heard of graphs—not the $y = x^2$ kind, but networks of vertices and edges that model

friendships, subway systems, computer networks, and much more. In graph theory textbooks, graphs are studied as abstract combinatorial objects. But here's a more visual (and slightly artistic) question:

Can you draw a graph nicely?

What does "nicely" even mean? No crossings? Straight edges? Small area? Symmetry?

In this class, we turn aesthetic questions into precise mathematics. Once a graph has a "nice" drawing, even more questions arise. How can we find such a drawing among the millions of possible ways to draw it? Can we efficiently check whether a graph can be drawn nicely at all? And if we take the dark side—how bad can a drawing possibly be?

Homework: Recommended (with some challenging extension problems for those who want them).

Class format: Mostly lecture, with interactive discussions and exploratory problems each day.

Prerequisites: Basic graph theory: what graphs, vertices, edges, and degrees are; the handshaking lemma; and notation such as K_n and $K_{m,n}$.

Commutative algebra/algebraic geometry (Week 1/2) (🌶🌶🌶 → 🌶🌶🌶🌶, Mark, TWΘFS)
In its classical form, algebraic geometry is the study of sets in n-dimensional space that can be described by polynomial equations (in n variables). This is both a very old and a quite active branch of mathematics, and for over a century now it has relied heavily on commutative algebra — that is, on the properties of commutative rings and related objects. We'll start by looking at some of those, including prime and maximal ideals and a review of quotient rings, and we'll see how the algebra can be used to give us information about the geometric sets. For instance, we'll use the algebra to show that if a set can be given by polynomial equations, then a finite number of such equations will do. We may also see how to translate the idea of dimension into the language of algebra. There may well be cameo appearances by the axiom of choice (in the guise of Zorn's lemma) and a bit of point-set topology (on a space whose points are ideals!), but you don't need to know any of those things going in. In the second week of the class, I hope, among other things, to prove Hilbert's famous Nullstellensatz ("Theorem of the Zeros"), arguably the starting point for modern algebraic geometry, at least for the case of two variables. (The theorem will presumably be stated and used in the first week.)

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Basic familiarity with rings (including polynomial rings) and fields. It will help if you have seen quotient rings, but they will be covered briefly.

Introduction to Linear Algebra (🌶🌶, Glenn, TWΘFS)
Linear equations are used across mathematics, science, and engineering. In pure mathematics, calculus is approximation by linear equations, directed graphs often describe linearly evolving processes, and you'll see many more applications in future courses here at camp. In computer science, linear equations are the primary ingredient in machine learning models, and also help render 3D objects onto a 2D screen in games and other software. In physics, "quantum superposition" just refers to the fact that particles are described by linear equations, and in engineering, giant systems of linear equations are used to simulate and design projects.

The study of linear equations and linear processes is called *linear algebra*. They are (relatively) easy to solve and analyze, and thus most other mathematical fields routinely use linear algebra to solve more complex problems. You may think that you already learned about linear equations in middle school or high school Algebra 1, but we'll learn so much more. By the end of this course, you will understand:

- How computers solve systems with millions of equations
- The geometry of n-dimensional space and why that is useful

- How to derive that the n^{th} Fibonacci number is $\frac{1}{\sqrt{5}}\left(\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right)$, and general limiting behavior of linear processes
- How JPEG compresses images with a new coordinate system, and why it's important the axes stay perpendicular to each other
- . . . and so much more!

Homework: Required

Class format: Interactive lecture

Prerequisites: None

QR codes, Polynomials, and Finite Fields (🌶🌶🌶, *Eric*, TWΘFS)
QR codes are the familiar 2-dimensional patterns like

often used to share links or data electronically. These are examples of error-correcting codes: if some of the image gets scrambled you can still accurately decode the intended message! This class is about the mathematics behind these codes, which it turns out is basically just polynomials. Specifically we'll cover Reed–Solomon error correcting codes, the finite field with 256 elements, and the algorithms for encoding and decoding data in QR codes, on top of learning a little about the history and non-mathematical aspects of QR codes. Using our powers we'll also learn a trick for embedding images in QR codes, like

Homework: Recommended

Class format: We'll have a mixture of lectures and worksheets

Prerequisites: Basic linear algebra, at the level of knowing how to represent and solve systems of linear equations using matrices, and have heard the phrase "dimension of a vector space" before. Some basic number theory is helpful. If you've seen the Euclidean algorithm for computing greatest common divisors, and you know that a is invertible mod n iff $\gcd(a, n) = 1$ that should be enough.

10:10 AM Classes

Everyone WILL Hate Analysis (🌶🌶, Ben, TWΘFS)
There are a lot of things that your first-time calc teacher didn't tell you about—because they shouldn't have. But they could have! If they were going a little bit off of the "standard path," that is. This course is about that stuff. We'll see all kinds of assorted bad behavior, such as differentiable functions with derivatives that aren't continuous, functions that satisfy the conclusion of the intermediate value theorem, but aren't continuous, and we'll talk about what mathematicians mean by "increasing" and why it's *slightly* different than "the derivative being positive." After that, we'll learn about "smooth functions," which sound like they should be nice, based on their name! . . . And they mostly are.

Mostly.

We'll also exhibit a few nice proofs of calc-related-results, including a lot of ones that are well known, like L'Hôpital's Rule, and one or two that are more obscure.[1]

Homework: Recommended

Class format: Mostly interactive lecture

Prerequisites: You should be familiar with derivatives and the interpretation of derivatives, as covered in standard calculus courses

Four Color Theorem (🌶🌶🌶, Misha, TWΘFS)
When you color a political map, you would like to give regions that share a border different colors, so that the boundary between them is easy to pick out. With the advent of Google Maps, there's not a

[1] I'd drop a name here, but name-dropping an obscure theorem seems kind of like an exercise in futility.

lot of money in cartography, so you'd like to save a bit of it by using as few colors as possible. How low can you go?

It is not hard to find maps that require four colors. It was observed in 1852 (and possibly before then, but not to mathematicians) that no maps have been found which need more colors than that. Proving this became the celebrated Four Color Problem and motivated the development of much of modern graph theory.

It was not until 1976 that a proof was found! Appel and Haken's proof was a novelty for the time: it had so much casework that a computer was necessary to check all of it. You may have noticed recently that people have feelings about computers doing math; this was even more true in 1976.

The goal of this class is to understand how the proof works, what it was that the computer did, and why we should (probably) trust it.

Homework: Recommended

Class format: Lecture

Prerequisites: None. We will follow the early 20$^{\text{th}}$ century graph theorists and do graph theory without using the word graph or even necessarily knowing what a graph is.

Hyperbolic Geometry (🌶🌶, Arya, TWΘFS)
What is geometry, really? What's so special about "Euclidean geometry" that mathematicians since Pythagoras and Euclid (circa many years BC) believed this to be the natural geometry to study? It turns out that Euclidean geometry is indeed the "natural" geometry to consider, if the underlying space is an infinite flat plane. If the underlying surface is curved or shaped differently, the geometry changes. As a simple example, in Euclidean geometry, there exists a unique straight line between any two points, which is also the shortest path between the two points. However, if you're living on a sphere, then there are infinitely many shortest paths between the north pole and the south pole (indeed, any direction you start will give you a shortest path).

In this class, we shall learn about non-Euclidean geometry - various geometric models that differ from Euclidean geometry in certain aspects. We will specifically focus on hyperbolic geometry after the first couple of days. There are a few reasons why hyperbolic geometry is particularly nice. For one, "most spaces" will have their "natural geometry" be hyperbolic. Secondly, there are strong connections between hyperbolic isometries, number theory and algebra. Thirdly, we can understand a lot by studying the "boundary at infinity", a feature allowed by hyperbolic spaces.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: For one of the days, it will be useful to know how to multiply 2×2 matrices. Other than that, no prereqs.

Introduction to Number Theory (🌶🌶🌶, Mark, TWΘFS)
How do you find $\gcd(a, b)$ for two large integers a and b without having to factor them? Which integers are the sum of two (or the sum of three, or the sum of four) perfect squares? What postages can you get (and not get) if you have only 8 cent and 17 cent stamps available? Besides the famous 3,4,5 triangle (and scaled versions of it), what right triangles are there for which all the side lengths are integers? How does the RSA algorithm (used for such things as sending confidential information, such as your credit card number, over the internet) work? (If you know the answers to *all* these questions, please don't take this class; you'll be bored, and you might make others feel bad.) Besides the answers to such questions, number theory offers insight into many beautiful and subtle properties of our old friends, the integers. For thousands of years professional and amateur mathematicians have been fascinated by the subject (by the way, some of the amateurs, such as the 17$^{\text{th}}$ century lawyer Fermat and the theoretical physicist Dyson who passed away in 2020, are not to be underestimated!) and

chances are that you, too, will enjoy it quite a bit. Although we'll start from scratch, in order to touch on as many as we can of the topics mentioned above (and maybe a few others) the class will go at a good pace—thus the three chilis.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Nothing beyond modular arithmetic (which I can catch you up on individually if needed)

Polynomial Method in a Hat Puzzle (🌶🌶, Nikita, $\boxed{\text{TW}}\Theta\text{FS}$)
There are 100 sages standing in a circle, and each is given a hat of one of seven colors. The colors are chosen independently and uniformly at random. Each sage can see the hats of all the other sages, but not their own.

Before the hats are assigned, the sages may agree on a strategy. Starting with a fixed first sage and proceeding clockwise, each sage in turn either says, "Pass," or guesses the color of their own hat. The process may continue for several rounds around the circle, with sages passing repeatedly, but it ends as soon as someone makes a guess.

The sages pass the test if the first sage to make a guess guesses their own hat color correctly. What is the maximum possible success probability?

This puzzle has a solution considering polynomial hypersurfaces in $\mathbb{R}^{100}$. We will dig into this solution and will get for free a solution of a problem from Vojtěch Jarník International Mathematical Competition none of the participants solved. But it's not that hard, I promise!

Homework: Recommended

Class format: Problem solving + Lecture

Prerequisites: Knowing what is the graph of $x^2 + y^2 - z^2 = 1$

Visualizing the 3-Sphere: The Hopf Fibration (🌶, Laithy, $\text{TW}\boxed{\Theta\text{FS}}$)
How can we visualize a sphere that lives in four-dimensional space? In this class, we will explore one of the most beautiful answers: the Hopf fibration.

The Hopf fibration gives a way to decompose the mysterious 3-sphere into a family of circles, with one circle for each point on an ordinary 2-sphere. In other words, it allows us to think of S^3 as being filled by circles indexed by points of S^2. This already sounds surprising, but the real magic is in how those circles fit together.

They are not arranged like a boring product $S^2 \times S^1$, where each circle sits independently over a point of the sphere. Instead, when we project the picture into ordinary 3-dimensional space, the circles appear linked with each other. In fact, every pair of circles is linked exactly once.

Along the way, we will use complex numbers, stereographic projection, and the Riemann sphere to build visual models of S^3.

Homework: Recommended

Class format: Interactive

Prerequisites: Comfort with complex numbers

11:00AM

Fundamentals (🌶–🌶🌶🌶🌶, Narmada, $\boxed{\text{TW}\Theta\text{F}}\,\text{S}$)
Do you want to get practice with

- working with mathematical formalisms,
- identifying and writing correct proofs, or
- working with common mathematical concepts?

For one week only[2], we will have worksheets available for you during TAU to practice problems of varying difficulty levels. Each day is somewhat independent of the other ones, although there will be some continuity among the problems.

- T: Sets and quantifiers;
- W: Proving and negating statements;
- Θ: Functions;
- F: Induction.

If you are not familiar with any of the material, there will be a short 'lecture' at the beginning of TAU 1 to define that day's terminology. Note that this starts at 11:00, *not* at 11:10. Let's put the FUN in Fundamentals!

Homework: Recommended

Class format: Worksheet

Prerequisites: None

1:10 PM Colloquia and Events

Hypnotic Stylings of Aperiodic Tilings (Zach, Monday)
Structure in chaos, simplicity in complexity, and periodicity in—wait, nevermind, these tilings are decidedly not periodic. That's the whole point! We'll investigate how some sets of shapes can tile the plane but *never* in a repeating way, as well as multiple strategies for proving this perplexing property. Look forward to dozens of dazzling diagrams and an array of laser-cut tiles to play with.

Buridan's Principle (Nikita, Tuesday)
Aristotle laughed at the idea that "the man who is violently, but equally, hungry and thirsty, and stands at an equal distance from food and drink, [. . .] therefore must remain where he is." We will discuss what modern math adds to this argument. However, how much time would we spend placing the man in the middle of the food–drink segment? Will it actually happen or will it only happen once a monkey randomly types *Hamlet*?

The Mathematics of M.C. Escher (Purple, Wednesday)
M.C. Escher created beautiful illustrations of impossible objects. But how is it possible to depict something impossible? The answer has a lot to do with one of the major themes of modern mathematics: understanding when, how, and why local phenomena fail to extend to global ones. Our goal in this talk is to explore this connection, ultimately building a machine, called cohomology, which produces a kind of measurement of the impossibility of figures like Escher's.

General Relativity and Black Holes (Laithy, Thursday)
Einstein's theory of general relativity is a theory of spacetime: it interprets gravity not as a force pulling objects through space, but as the curvature of spacetime itself. In this theory, the universe is modeled by a "curved" spacetime satisfying "Einstein's equations". One of the central problem in mathematical physics is to understand the possible solutions of these equations and what they predict about our universe.

In this talk, we will explore one of the strangest and most striking predictions of general relativity: black holes and white holes. One of the first exact solutions of Einstein's equations is the Schwarzschild solution, which has the remarkable property that it contains a region from which not even light can

[2] This is a lie. You can pick up worksheets any time during camp!

escape. Even more strangely, its maximal extension contains two identical copies of exterior universes, a black hole region, a white hole region, and a special sphere that links both universes.

To make these ideas precise, we will develop some of the basic machinery of Lorentzian geometry and spacetime compactification, which allow us to define black and white holes rigorously. We will then see that black hole formation can sometimes be predicted from a single slice using the notion of a "trapped surface". Finally, we will end with a glimpse of Christodoulou's celebrated work on the formation of black holes, which shows that this phenomenon is not just an artifact of one special exact solution, but arises in a broad families of solutions to Einstein's equations.

Imposter Phenomenon (The Staff, Friday)
Imposter phenomenon is an experience which many of us in the math community share. The goal of this event is for us as a community to discuss these experiences and how we can support ourselves and each other through them. There will be a post-lunch dessert, so come by for that!

2:10 PM Classes

Mapping Class Groups (🌶🌶🌶🌶, Arya, TWΘFS)
A "surface" is a space which locally looks like a disk at every point. Common examples of surfaces are a cylinder, a sphere or a torus. For any surface, a natural object of study is the group of "homeomorphisms" (maps that preserve the topology of the surface) of the surface to itself. This group, however, is EXTREMELY large. We instead study the group of homeomorphisms of the surface to itself *up to "homotopy"* (continuous deformations of the surface without cutting and gluing). This is a discrete group, with a well-defined algebraic structure and is sufficiently rich to recover information about the geometry of the surface and dynamics of the homeomorphisms in question.

In this class, we shall study mapping class groups of surfaces, and study how their algebraic structure tells us information about the surface.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: We will assume familiarity with matrices and linear transformations of $\mathbb{R}^2$, and assume the definitions of groups, subgroups and group actions. Some familiarity with topology would be helpful.

How to Discover the Bell Curve (🌶🌶🌶, *Apurva Nakade*, TWΘFS)
On January 1, 1801, the first asteroid, Ceres, was discovered, and just a month later, it was lost again behind the glare of the sun. Astronomers faced the challenge of calculating Ceres' orbit months into the future using only 22 observations. Being the first object of its kind to be discovered, astronomers did not know what eccentricity its orbit should have. Many tried guessing based on their "expertise," but Gauss solved the problem purely empirically—by analyzing data and making no further assumptions beyond Kepler's laws.

Gauss ended up with the best estimate—the asteroid was rediscovered exactly where he predicted—and became the OG data scientist. In the process, he invented the method of maximum likelihood estimation, (not so) ordinary least squares, applied it to linear regression, and discovered his most enduring legacy—*the bell curve* (and some astronomy stuff).

In this class, we will learn all of these things (not the astronomy stuff).

Homework: Required

Class format: The class will be a mix of lectures and problem solving worksheets.

Prerequisites: You should know how to find derivatives and optimize functions using the first/second derivative tests. This is the only serious prerequisite. You might also need to compute simple integrals, but nothing too crazy.

Even though this class is listed under probability, knowledge of probability theory is not required.

Introduction to Group Theory (🌶🌶, Neelam, TWΘFS)
Have you ever wondered how a speedcuber can solve a completely scrambled Rubik's Cube in under five seconds? Or how your phone keeps your private messages safe from hackers? Then group theory is the math superpower you're looking for. In a standard algebra class, you spend a lot of time looking at numbers and equations. In this week-long crash course, we are throwing out these numbers and looking at the underlying structure.

Group theory is the study of symmetry, patterns, and the rules that govern how things can combine and change. Over the course of five days, we will dive into the hidden structures that control many games and are foundational to modern communication or technology.

Puzzle Mastery: Discover how groups hold the secret to conquering the Rubik's Cube and cracking sequential strategy games like the Towers of Hanoi.

The Science of Secrets: Peek behind the curtain of modern cybersecurity to see how encryption and decryption use abstract patterns to keep data safe from digital spies.

Symmetry Unlocked: Learn to see the beautiful, invisible geometry found in math and nature.

Who is this for? If you like solving puzzles, logic games, or breaking codes, you already think like a group theorist. Come find out how deep the rabbit hole goes!

Homework: Recommended

Class format: Lecture+Exercises+Small Homework

Prerequisites: None.

Proving Puzzles Hard (🌶🌶, Della, TWΘFS)
If you've solved logic puzzles,[3] you might have noticed they can be pretty hard. But what does that mean, mathematically? And how can we prove it?

We'll talk about one answer (called "NP-hardness"), from computational complexity theory, and a handful of tools designed specifically for applying this concept to logic puzzles. By the end of the class, you'll be able to prove a logic puzzle hard by drawing just one diagram!

Homework: Recommended

Class format: Lecture

Prerequisites: None

The Lebesgue Integral (🌶🌶🌶🌶, Riley, TWΘFS)
Much like a line of infinite mathematicians at a café where each orders half as much coffee as the person ahead of them, the Riemann integral doesn't know its limits. In particular, there is a sequence of bounded Riemann-integrable functions defined on $[0, 1]$ with integral 0 that increases and converges pointwise to a bounded function that is not Riemann-integrable. This should make you a) sad about being hamstrung analytically and b) unable to sleep at night when you switch an integral and a limit, despite the best efforts of the many instructors who may pat your head and encouragingly cheer, "You can do it!" Luckily, the Lebesgue integral, much like a certain putatively perfumed air, is there to provide respite, respite and nepenthe from this troubling news. Indeed, when instructors neglect to justify switching a limit and an integral, the dominated convergence theorem often does the trick. In

[3] Think Sudoku, not "the cat owner lives next to the blue house".

this class we aim to define the Lebesgue integral and prove its related convergence theorems. This requires that we introduce and develop the Lebesgue measure, which allows us to assign "lengths" or "masses" to many (but not all, if you are an axiom of choice enjoyer) subsets of the real numbers.

Homework: Required

Class format: Lecture

Prerequisites: Things I will assume you have some familiarity with: epsilons-delta arguments, infima and suprema, infinite unions and intersections, pointwise versus uniform convergence, the rationals being dense in the reals, countably infinite versus uncountably infinite sets.

Things that are helpful to know: what an open and closed set are (in $\mathbb{R}$ at least), that the uniform limit of a sequence of continuous functions is continuous, a definition of the Riemann integral (one that deals with well-definedness), liminfs and limsups.

3:10 PM Classes

Concentration (Week 1 of 2) (🌶🌶🌶, Misha, TWΘFS)
Some people have an easier time focusing on class than others. This is called a concentration inequality.

In this class, we will study some famous concentration inequalities. We will use them to prove that various random variables are usually very close to their average values. Typically, "usually" and "close" will have asymptotic meanings we will represent with symbols such as O, Ω, and Θ, though by the end of the first week, I hope to show you a random variable which is concentrated with high probability on just two values.

Homework: Recommended

Class format: Lecture

Prerequisites: "If $\Pr[\mathbf{X} = k] = \binom{n}{k} p^k (1-p)^{n-k}$ for $k \in \{0, 1, \ldots, n\}$, then $\mathbb{E}[\mathbf{X}] = np$." You do not need to know a proof of this statement, but the more comfortable you are reading it, the more prepared you are for concentration.

Here is a list of the Greek letters you'll encounter: α (alpha), Δ (Delta), ε (epsilon), Θ (Theta), μ (mu), π (pi), σ (sigma), χ (chi), Ω (Omega).

Hairy Ball Theorem (🌶🌶🌶🌶, Laithy, TWΘFS)
Can you comb all the hairs on a perfectly round sphere so that every hair lies flat and the combining changes smoothly from point to point? Surprisingly, the answer is no: no matter how clever you are, somewhere on the sphere you are forced to create a cowlick. This strange fact is actually a famous theorem in topology called "the hairy ball theorem".

We will start by making precise what it means to "comb" a sphere using tangent arrows drawn on its surface that we will call "vector fields". We will discover that a perfect smooth combing would allow us to smoothly deform every point of the sphere to its opposite "antipodal" point in a very special way. The heart of the class is understanding why that deformation is impossible. Surprisingly, proving this requires us to build some powerful machinery from differential topology, such as homotopies, Sard's theorem, orientation and degree of smooth maps.

We will also compare spheres of different dimensions: it turns out that even-dimensional spheres cannot be combed, whole odd-dimensional spheres can. If time permits, we will also discuss the Poincaré-Hopf theorem, which explain the Hairy Ball theorem as part of a much broader story about how the shape of a space controls the behaviour of vector fields on it.

Homework: Recommended

Class format: Interactive lectures

Prerequisites: You should be comfortable with vectors, dot products, matrices, and the derivative of functions on $\mathbb{R}$. Some linear algebra will be helpful, especially what it means for a linear map to be onto.

Math = Computer Science (🌶🌶, Purple, TWΘFS)
The modern subjects we call mathematics and computer science both came out of the same sets of debates around the turn of the 20th century, around the nature of logical reasoning. Today they seem very different, but these historical similarities show up in a very fundamental way: the Curry–Howard isomorphism, which unifies logic with programming languages. In our journey to prove this fact, this course will touch on questions like:

- How do we mathematically reason about formal logic?
- How might we try to automate verification of mathematical proofs?
- How can we use mathematical ideas to reason precisely about the behavior of computer programs?

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

Problem-Solving: Triangle Geometry (🌶🌶 → 🌶🌶🌶, Zach, TWΘFS)
Come explore the rich, diverse, and endlessly surprising world of triangle geometry! Triangles have loads of named "center" points, and we'll venture well beyond the classical centroid and orthocenter into some lesser-known gems. Why has the symmedian point been called "one of the crown jewels of modern geometry"? Why is the existence of Feuerbach's point even reasonable (I'm still not convinced...), and how might we approach its construction synthetically (i.e., without inversion)? What are the (literally!) more than $10,000$ triangle centers listed in the Encyclopedia of Triangle Centers, and how can this encyclopedia be interpreted?

This class is largely problem based: there will be some lecturing, but much of the time you will present your solutions to the previous day's olympiad-style homework problems.

Homework: Recommended

Class format: Some lecture, some problem solving time, some camper presentations

Prerequisites: None.

Voting Theory: The Problem (🌶, Ben, TWΘFS)
In 2023[4], Burlington, VT adopted a "ranked-choice" voting system[5], under which voters can submit a ranked list of candidates they support (their favorite, then their next favorite, and so on). If their favorite candidate gets the fewest votes of anyone, their ballot gets counted for their next favorite, and so on and so forth. You can read more about this e.g. at https://www.burlingtonvt.gov/191/Ranked-Choice-Voting.

In a lot of the rest of Canada/USA, the electoral system is a simpler one in which everyone votes for *one* candidate, and whoever gets the most votes wins. This one sounds nice, until you start thinking and realize that if two broadly similar candidates are running, they can "split votes" so that neither of them wins; their supporters should "vote strategically" to increase the chances of *one of them* winning. Instant runoff systems, like Burlington's, make it easier for voters to avoid this kind of headache.

However, there is a *slightly* over-exuberant claim that gets made about instant runoff systems, which is that "you don't have to vote strategically in these systems." This claim is—unfortunately—not true.

[4]Right before the *last* time that camp was here, by the by

[5]Strictly speaking, Burlington's method is called an "instant runoff."

But the *reason* it's not true is very interesting! There's a theorem that says that *any* reasonable election system that looks *anything like* ranked-choice voting will have some annoying "edge cases" in which some voters should be voting strategically.[6]

This class aims to prove that theorem. I'll also remark, at this point, that this theorem is *not* Arrow's Impossibility Theorem—we will, in fact, prove Arrow's Theorem only because it is a helpful tool for proving our main result, the Gibbard–Satterthwaite Theorem.

Homework: Recommended

Class format: Lecture

Prerequisites: None

[6] Terms such as "reasonable," "election system," "anything like," and "strategically," may need to be defined a bit more precisely.