Contents

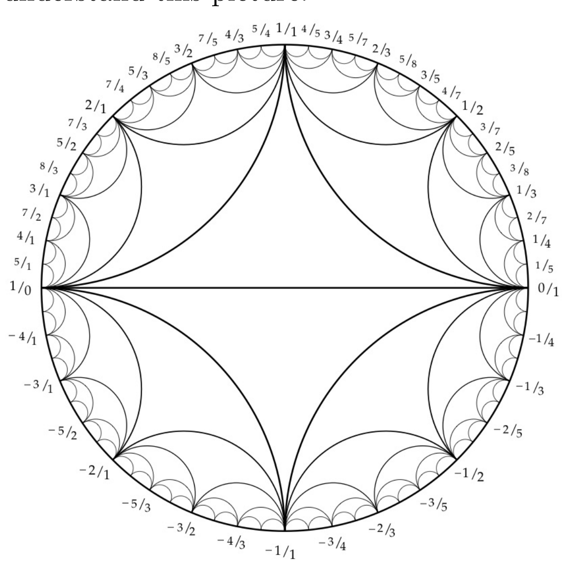
9:10 Classes

Electrifying Random Trees. (Will, Tuesday–Wednesday)
Consider a graph, and a particular edge in this graph. A natural question to ask is: how important is this edge in connecting the graph together? Here are two ways we might quantify this importance:

(1) The spanning trees of the graph give all the ways of minimally connecting the vertices using a subset of the edges. So look at all the spanning trees, and figure out how many of them contain our edge. Put another way, what is the probability that a random spanning tree contains the edge?
(2) Build your graph out of 1-ohm resistors. Hook up a 1-volt battery to the ends of your edge, and then measure how many amps of current are passing through that edge (as opposed to going through other paths in the graph).

These two numbers are actually the same! In this class, we'll see why. On the way, we'll see how one might generate a random spanning tree in the first place.
Chilis:
Homework: Recommended.
Prerequisites: Familiarity with the language of graphs (i.e., what a tree is; what the degree of a vertex is).

Farey Tales. (J-Lo, Tuesday–Friday)
The goal of this course is to understand this picture:



Along the way we will encounter Farey series, the Euclidean algorithm, and a geometric interpretation of continued fractions. But really it's all about the picture.

Chilis:

Homework: Recommended.

Prerequisites: Basic theory of 2×2 matrices (multiplication, inverses, determinant, how they act on vectors).

Galois Theory Crash Course. (Mark, Tuesday–Friday)
In 1832, the twenty-year-old mathematician and radical (in the political sense) Galois died tragically, as the result of a wound he sustained in a duel. The night before Galois was shot, he hurriedly scribbled a letter to a friend, sketching out mathematical ideas that he correctly suspected he might not live to work out more carefully. Among Galois' ideas (accounts differ as to just which of them were actually in that famous letter) are those that led to what is now called Galois theory, a deep connection between field extensions on the one hand and groups of automorphisms on the other (even though what we now consider the general definitions of "group" and "field" were not given until fifty years or so later). If this class happens, I expect to be rather hurriedly (but not tragically) scribbling as we try to cover as much of this material as reasonably possible. If all goes well, we might conceivably be able to get through an outline of the proof that it is impossible to solve general polynomial equations by radicals once the degree of the polynomial is greater than 4. (This depends on the simplicity of the alternating group, which we won't have time to show in this class but which *may* be shown in a separate week 5 class.) Even if we don't get that far, the so-called Galois correspondence (which we should be able to get to, and prove) is well worth seeing!

Chilis:

Homework: Recommended.

Prerequisites: Group theory; linear algebra; some familiarity with fields and with polynomial rings.

Packing Permutation Patterns. (Misha, Tuesday–Wednesday)
Prepare by picking a permutation π and a pattern P. Probabilistically pick $|P|$ pieces of π: perhaps putting them together produces P? Let $\rho_P(\pi)$ be the probability of producing P.

To *pack* P in π, puff up this probability, making P as plentiful as possible. We will ponder the packing problem for $P = 132$ (and plenty of its pals) using a progression of powerful problem-solving procedures.

(For returning campers: this material is a subset of my class on flag algebras last year.)

Chilis:

Homework: Recommended.

Prerequisites: None.

The Polish Attack on the Enigma. (Ben, Thursday–Friday)
In the early 1930s, the German Enigma machine seemed to offer a great deal of security, due to the sheer number of possible setups for the machine. However, Polish intelligence realized that the code could be broken, using group theory.

We'll see how the Polish attack on the early Enigma worked, which is both a historically interesting topic and a testament to the power of group theory.

Chilis:

Homework: Recommended.

Prerequisites: Group theory.

Young Tableaux and Probability. (Shiyue and Andrew, Thursday–Friday)
Guess who is super excited about Young tableaux in MC2019? In this course, we will give a different proof for the Hook Length formula, using probability. Maybe in Week 2, you have seen that the Hook Length Formula gives us a way of counting the number of standard Young tableaux, which turns out to be the dimension of irreducible representations of the symmetric group S_n. In Week 3, you might have seen a complicated proof of the Hook Length Formula which required building up several identities involving generating functions. In this course, we will use probability to think about hook walks and the recursive relations between standard Young tableaux and their sub-standard Young tableaux (which is a very natural idea when one tries to build up a standard Young tableaux). Eventually, we will give an elegant derivation of the Hook Length Formula.

Chilis:

Homework: Recommended.

Prerequisites: Induction, some knowledge about probability will be helpful but not necessary.

10:10 Classes

A Proof of the Sensitivity Conjecture. (Tim!, Thursday–Friday)
The Sensitivity Conjecture was a big open problem in theoretical computer science for thirty years. I spent some time thinking about it earlier in grad school. I would have offered to show you the proof earlier in camp (you know, back when I was at camp), but when we were making the four-week schedule, there was no proof yet (as far as the world knew)! In fact, Hao Huang posted his proof during Week 2 of camp. The proof is amazingly short, tantalizingly simple and very beautiful, yet also clever enough that it eluded prominent mathematicians for decades. It uses ideas from graph theory and linear algebra, by looking at eigenvalues.

More about the conjecture itself: The Sensitivity Conjecture is (or I should say, *was*) a conjecture about the complexity of boolean functions. Theoretical computer scientists care a lot about boolean functions (in part) because computers operate with boolean values. They care about complexity because they care about how complicated their programs are (in various senses of the word "complicated" — how long they take to run, how much memory they use, etc.).

There are many ways to measure complexity. Many of these complexity measures are known to be equivalent (or more precisely, *polynomially related*) to each other; these include *degree*, *(deterministic) decision tree complexity*, *bounded-error randomized decision tree complexity*, *certificate complexity*, *block sensitivity*, *approximate degree*, and *quantum decision tree complexity with bounded error*. This is both surprising (because these complexity measures have very different-sounding definitions) and a relief (because if all these complexity measures disagreed with each other, then who would you trust?). However, there was one complexity measure — called *sensitivity* — that was not known to be in the club. The Sensitivity Conjecture asserted (sensibly) that sensitivity is polynomially related to all those other complexity measures.

It seems fortuitous that this problem, which I've spent time on and which has been open for so long, was solved during Mathcamp and has such an elegant proof. If I were still at camp, I'd definitely propose a Week 5 class on it. Right now, though, I am in the Midwest, land of cows and rolling cornfields. But you know what… someone hold my cheese; I'm on my way!

In this class, we'll follow the story of the Sensitivity Conjecture from its development up to and including its proof.

Chilis:

Homework: Optional.

Prerequisites: You should know what *degree* means in graph theory, and what an *eigenvalue* is in linear algebra.

Calculus without Calculus. (Tim!, Tuesday–Wednesday)
If you've taken a calculus class in school, you've surely had to do tons and tons of homework problems. Sometimes, calculus knocks out those problems in no time flat. But other times, the calculus solution looks messy, inelegant, or overpowered. Maybe the answer is nice and clean, but you wouldn't know it from the calculation. Many of these problems can be solved by another approach that doesn't use any calculus, is less messy, and gives more insight into what is going on. In this class, you'll see some of these methods, and solve some problems yourself. Some example problems that we'll solve without calculus:

- Lizka is 5 cubits tall and Eric is 3.9 cubits tall, and they are standing 3 cubits apart. You want to run a string from the top of Lizka's head to the top of Eric's head that touches the ground in the middle. What is the shortest length of string you can use?
- Assaf rides a bike around an elliptical track, with axes of length 100 meters and 150 meters. The front and back wheels (which are 1 meter apart) each trace out a path. What's the area between the two paths?
- A dog is standing along an inexplicably straight shoreline. The dog's person stands 20 meters way along the shoreline throws a stick 8 meters out into the water. The dog can run along the shoreline at 6.40 meters per second, and can swim at 0.910 meters per second. What is the fastest route that the dog can take to get to the stick?
- Where in a movie theater should you sit so that the screen takes up the largest angle of your vision?
- What's the area between the curves $f(x) = x^3/9$ and $g(x) = x^2 - 2x$?

Amaze your friends! Startle your enemies! Annoy your calculus teacher!
Chilis: 🌶🌶
Homework: Recommended.
Prerequisites: Some calculus will be useful for context, but we won't actually use calculus (that's the point).

Conway–Coxeter Frieze Patterns. (Véronique, Tuesday–Wednesday)
Conway and Coxeter introduced frieze patterns in 1973: a frieze pattern over a field K is a map that associates to specific points in $\mathbb{Z}^2$ elements of K following a given rule. In this class, you will study properties of frieze patterns (periodicity, construction from a given diagonal, construction from a given row). More important, you will study the connection between frieze patterns and triangulations of an n-gon. Indeed, the number of triangles incident to each vertex appear in a frieze pattern, as well as a labeling of the vertices of the triangulated n-gon based on the triangles they share.
Chilis: 🌶
Homework: Optional.
Prerequisites: None.

Elegant Applications of Linear Algebra to Combinatorics. (Bill, Thursday–Friday)
In this class we'll use ideas from linear algebra to prove beautiful (and surprising!) theorems in discrete math. For example, we'll prove:

- For any irrational number x, it's impossible to tile a $1 \times x$ rectangle with finitely many squares (*even* if the squares are permitted to have both irrational and rational side-lengths).

The theme of the class will be that basic ideas in linear algebra (e.g., linear independence, dimension, bases) can be immensely useful when analyzing seemingly unrelated problems (e.g., tilings of a rectangle, or set systems satisfying certain intersection properties).
Chilis: 🌶🌶🌶
Homework: None.

Prerequisites: Basic familiarity with linear algebra. Understanding of terms: vector space, basis, dimension, linear function, linear independence. No familiarity with eigenstuff is required.

Factoring in the Chicken McNugget Monoid. (Gabrielle, Thursday–Friday)
When they were first released, Chicken McNuggets were sold in packs of 6, 9, and 20. The Chicken McNugget Monoid is the set of numbers of Chicken McNuggets that can be purchased (Chicken McNugget numbers), i.e. numbers of the form $6a + 9b + 20c$. Notice that this expansion is not unique: $18 = 6 \cdot 3$ and $18 = 9 \cdot 2$. This is a form of nonunique factorization!

We will analyze this problem, and related problems in numerical monoids, through the lens of nonunique factorization.

Chilis:

Homework: Recommended.

Prerequisites: None.

Morse Theory. (Kayla, Thursday–Friday)
Topologists like donuts. Suppose that we want to cover our donut in chocolate sauce when it is oriented vertically (so that the hole of the donut is perpendicular to the floor). What would the chocolate sauce function look like? Would this function have critical points? What does it mean to be a critical point of a chocolate sauce function?

Morse theory is a branch of topology that enables us to analyze the topology of a manifold by studying differentiable functions on that manifold. Moreover, some differentiable functions can tell us information about how to build a CW complex structure for our space!

Chilis:

Homework: Optional.

Prerequisites: Topology and calculus.

The Class Number. (Gabrielle, Tuesday–Wednesday)
Kummer was able to prove Fermat's Last Theorem in the case where n is a regular prime. In this class, we're going to figure out what that means and why that is useful.

Chilis:

Homework: Recommended.

Prerequisites: Intro to Algebraic Number Theory.

The Riemann Zeta Function. (Mark, Tuesday–Wednesday)
Many highly qualified people believe that the most important open question in pure mathematics is the Riemann hypothesis, a conjecture about the zeros of the Riemann zeta function. Having been stated in 1859, the conjecture has outlived not only Riemann and his contemporaries, but a few generations of mathematicians beyond, and not for lack of effort! So what's the zeta function, and what's the conjecture? By the end of this class you should have a pretty good idea. You'll also have seen a variety of related cool things, such as the probability that a "random" positive integer is not divisible by a perfect square (beyond 1) and the reason that $-691/2730$ is a useful and interesting number.

Chilis:

Homework: None.

Prerequisites: Single-variable calculus, including infinite series. Previous experience with functions of a complex variable may help a bit, but is *not* required.

11:10 Classes

A Very Difficult Definite Integral. (Kevin, Tuesday)
In this class, we will show

$$\int_0^1 \frac{\log(1+x^{2+\sqrt{3}})}{1+x}\,dx = \frac{\pi^2}{12}(1-\sqrt{3}) + \log(2)\log(1+\sqrt{3}).$$

We'll start by turning this Very Difficult Definite Integral into a Very Difficult Series. Then we'll sum it!

We'll need an unhealthy dose of clever tricks involving some heavy-duty algebraic number theory. It will be Very Difficult.

(This class is based on a StackExchange post by David Speyer.)

Chilis: 🌶🌶🌶🌶

Homework: None.

Prerequisites: If you have taken all of the algebraic number theory classes at camp, you will recognize some of the ingredients in results that we will assume.

Calculus on Young's Lattice. (Kevin, Thursday)
Young's lattice is a beautiful poset capturing the structure of partitions, with connections to combinatorics, geometry, and representation theory. Young's lattice is almost unique among posets[1], with the remarkable property that we can do calculus on it. We'll discuss how, and we'll see how doing calculus on Young's lattice yields powerful enumerative results, including some famous identities involving Young tableaux. Plus, if you like exponentiating things, we'll even get a chance to exponentiate xD!

Chilis: 🌶🌶🌶

Homework: None.

Prerequisites: Differential calculus. Familiarity with partitions or any of Shiyue's Young Tableaux classes will be helpful.

Cauchy–Davenport with Combinatorial Nullstellensatz. (Bill, Wednesday)
The Cauchy–Davenport Theorem states that for a prime number $p > 2$, and for $A, B \subseteq \mathbb{Z}_p$,

$$|A + B| \geq \min(p, |A| + |B| - 1).$$

In this class, we'll prove the Cauchy–Davenport Theorem using a powerful technique known as Combinatorial Nullstellensatz, in which one carefully defines a polynomial whose set of zeros captures some combinatorial problem; and then one uses a (somewhat peculiar) theorem about the zeros of multivariate polynomials in order to extract properties of the original combinatorial problem.

Chilis: 🌶🌶🌶

Homework: None.

Prerequisites: None.

Finite-ness. (Susan, Friday)
What does it mean for a set to be finite? One possible definition is that the set can be put into bijective correspondence with a natural number. But is that really the definition we want? It seems kind of inelegant to just take all the finite sets we already know and say, "That's all, folks!" Here are a couple possible definitions of finite-ness that don't reference the natural numbers:

- **Dedekind's Definition:** A set S is finite if every injection $f : S \to S$ is a surjection.

[1] Morally, there's only one other such poset, and no other lattices! Conjecturally, at least.

- **Tarski's Definition:** A set S is finite if every total-order on S is a well-order.
- **Kuratowski's Definition:** A set S is finite if and only if it is an element of the smallest class of sets containing $\emptyset$ and all $\{s\}$ for $s \in S$ which is closed under unions.

All of these definitions are equivalent to the natural numbers definition… as long as you believe the Axiom of Choice. Otherwise, only one of them is. And also the proofs are surprisingly subtle. Wait… is finiteness hard?

Chilis:

Homework: None.

Prerequisites: None.

Infinitely Many Proofs of Infinitely Many Primes. (Eric, Tuesday)
I'll fit as many proofs as I can into one class of the fact that there are infinitely many prime numbers. Some will be straightforward, others will connect us to great open problems (the Riemann zeta function will appear!), others will use topology (??), and more!

Chilis:

Homework: None.

Prerequisites: Some proofs may require a small amount of knowledge about topology or rings, but nothing beyond the basic definitions.

Matroids and Greed. (Will, Thursday)
Kruskal's algorithm is a way of choosing, in a graph with weighted edges, a spanning tree of minimal weight. It's a very simple greedy algorithm: just keep adding the least-weight edge you can, subject to the condition that you don't form a cycle.

A matroid is a mathematical structure that captures the idea of "independent subsets of a set." Key motivating examples include linearly independent subsets of a finite set of vectors, as well as forests and trees in a graph.

And in fact, just as in the case of graphs, we can use a greedy algorithm to find minimum-weight independent subsets of any matroid. But more is true: matroids are actually precisely characterized by the fact that a greedy algorithm works in this way. In this class, we'll look at a few equivalent ways of defining matroids, and then see why the greedy algorithm property is one of them.

Chilis:

Homework: Recommended.

Prerequisites: Know what "linear independence" is. Know what a tree (in the context of graph theory) is.

Perfect Numbers. (Mark, Friday)
Do you love 6 and 28? The ancient Greeks did, because each of these numbers is the sum of its own divisors, not counting itself. Such integers are called *perfect*, and while a lot is known about them, other things are not: Are there infinitely many? Are there any odd ones? Come hear about what is known, and what perfect numbers have to do with the ongoing search for primes of a particular form, called Mersenne primes – a search that has largely been carried out, with considerable success, by a far-flung cooperative of individual "volunteer" computers.

Chilis:

Homework: None.

Prerequisites: None

Perspectives on Cohomology. (Apurva, Assaf, J-Lo, and Eric, Tuesday–Friday)
Cohomology is a useful mathematical tool that comes in a variety of flavours, and many of your staff friends use cohomology in their research. In this class we'll have a different staff every day talk about their favourite flavour of cohomology, what is special about it, what it is useful for, and how they think about it. The topics, in order:

- Cellular Cohomology 🌶🌶🌶 (Apurva)
- Čech Cohomology 🌶🌶🌶🌶 (Assaf)
- The Chow Ring 🌶🌶🌶 (J-Lo)
- Galois Cohomology 🌶🌶🌶 (Eric)

Chilis: 🌶🌶🌶–🌶🌶🌶🌶
Homework: Optional.
Prerequisites: Each lecture will aim to be mostly self-contained, but some familiarity with linear algebra will be useful for all of the lectures.

Rescuing Divergent Series. (Mark, Thursday)
Consider the infinite series $1-1+1-1+1-1+\dots$. What is its sum? Maybe $(1-1)+(1-1)+\cdots=0$, maybe $1-(1-1)-(1-1)\cdots=1$. At one time mathematicians were quite perplexed by this, and one even thought the issue had theological significance. Now presumably it's nonsense to think that the "real" answer is $\frac{1}{2}$, just because the answers 0 and 1 seem equally good, right? After all, how could the sum of a series of integers be anything other than an integer?
Chilis: 🌶
Homework: None.
Prerequisites: A bit of experience with the idea of convergence.

The Good and Bad Mathematics of Pennsylvania's Gerrymandering Lawsuit. (Mira, Wednesday)
Partisan gerrymandering has never been successfully challenged in US federal courts, and with the Supreme Court's most recent decision, pretty much all hope is gone.

On the other hand, in 2018, the Pennsylvania Supreme Court declared that Pennsylvania's gerrymandered map violates the Pennsylvania Constitution's guarantee of "free and fair elections". Since many other states have similar guarantees, the focus of anti-gerrymandering activism looks like it's going to switch to state courts.

That all sounds promising, except for one thing. What the PA Court called the "most compelling" argument in the case was based on completely bogus statistics. There was also a really good statistical argument presented to the court, but the judges didn't seem to understand it. (The good argument, made by mathematicians, was complicated; the bad one, made by political scientists, had nice pictures.)

In this class, we'll look at some of the original expert witness testimony presented before the court, the original research papers that both arguments are based on, the opposing side's attempted rebuttal, and the judges' reactions. You will get a sense of both how important and how difficult it can be to communicate mathematical concepts to non-mathematicians. You will also be in a better position to evaluate similar anti-gerrymandering arguments going forward; there are likely to be many in the next few years, and the popular press is likely to make a complete mess out of them.
Chilis: 🌶🌶
Homework: Optional.
Prerequisites: None. (In particular, you can take this whether or not you took Assaf's Week 1 class.)

The Mathematical ABC's. (Susan, Wednesday)
Ever noticed how no one ever uses "d" as a variable in a calculus class? Or how "i," "j," and "k" are perfectly fine natural numbers if you're doing combinatorics, but not if you're studying noncommutative rings? In this class we'll go over the alphabet from A to Z, and talk about how to use (or not to use) these letters we know and love. If we have time, maybe we'll learn Mathematical Greek!
Chilis:
Homework: None.
Prerequisites: None.

Traffic. (Assaf, Tuesday)
As the saying goes, "you're not in traffic, you are traffic." Traffic is a game that almost[2] everyone plays. Everyone wants to be rational, but sometimes this rationality comes back and bites the collective. In this class, we'll explore scenarios where this effect happens. We'll look at Braess' Paradox, the Bus Motivation Problem, and spend some time discussing the formalism of congestion games.
Chilis:
Homework: None.
Prerequisites: None.

Why the Millennium Problems? (J-Lo, Friday)
With such a wide variety of unsolved problems out there, why did seven in particular stand out enough to counted as Millennium Prize Problems? In this class I will give a brief overview of the history behind each problem, and even though I won't be able to state all the problems precisely, I hope at least to convey a sense of the mathematical significance of each, and why people care about them.
Chilis:
Homework: None.
Prerequisites: None.

1:10 Classes

All Things Manifoldy. (Apurva, Tuesday–Friday)
Who said that mathematicians are not real doctors? We perform surgeries all the time. In this class, we'll take baby steps towards understanding manifolds. We'll learn some of the uber awesome techniques invented by topologists to study manifolds, perform surgeries on them, and do origami using simplices. By the end of the class you'll be able to visualize (some) manifolds in higher dimensions.

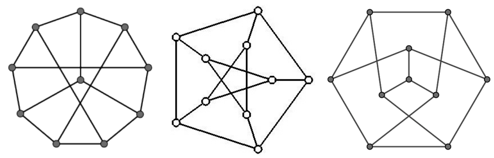
Incidentally, when Einstein tried to combine special relativity with Newton's gravity, nothing seemed to work. It took him a decade to finally realize a beautiful solution to the conundrum: our universe is a 4-dimensional manifold, and gravity is a measure of how the manifold curves. But what is a manifold?

The first two days are aimed at people who have never seen manifolds before. In the next two days we will cover Heegaard splittings and Dehn twists.
Chilis:
Homework: Recommended.
Prerequisites: None.

Crossing Numbers, from 1954 to 2012. (Marisa, Tuesday–Wednesday)
Suppose our goal is to draw graphs with as few edges crossing as possible. Take, for instance, the

[2] except for those annoying cyclists

Petersen graph. We can draw it in a nice way, displaying very pretty symmetry, with 6 edge crossings. (Terrible!) Or, exhibiting different symmetry, with five crossings. (Still terrible!) Or, exhibiting yet another symmetry, with three crossings. (Closer.) The best possible is actually two crossings; can you do it?



In this class, we'll prove nice bounds (and look for exact answers) for the minimum number of edge crossings of a graph, and we'll find very quickly that many questions about crossing numbers are wide open. In fact, even the crossing number of $K_{m,n}$ on the plane is open! (An answer to this question was conjectured—and a proof claimed—in the 1950s by Zarankiewicz, and was followed by a 1969 paper entitled "The decline and fall of Zarankiewicz's theorem.")

Chilis:

Homework: Recommended.

Prerequisites: None.

Generating-Function Magic. (Bill, Tuesday–Wednesday)

"A generating function is a clothesline on which we hang up a sequence of numbers for display."
– Herbert Wilf

The *generating function* of a number sequence $a_0, a_1, \ldots$ is the formal power series $\sum_{i=1}^{\infty} a_i x^i$. By encapsulating the number sequence within a single algebraic object, generating functions allow for us to derive results about the original number sequence in what are often highly unexpected ways.

In this class, we'll use generating functions to prove two of my favorite results:

- The number of integer partitions with odd-sized parts is equal to the number of integer partitions with distinct-sized parts. (We'll also see a beautiful bijective proof of this result!)
- The n^{th} Catalan number C_n, which counts the number of valid configurations of n pairs of parentheses, satisfies $C_n = \binom{2n}{n}/(n+1)$.

Chilis:

Homework: None.

Prerequisites: Students who have seen an example of generating functions used in the past will have an easier time, but I will make sure this is not necessary.

Going in Cycles. (Misha, Thursday–Friday)

A *knight* is a chess piece that jumps from a square to any other square exactly $\sqrt{5}$ units away. Put one of these in the corner of an 8×8 chessboard. Can it visit every other square of the board exactly once, then come back to the start?

This is an instance of the Hamiltonian cycle problem. In general, it's very hard to solve. We will talk about some ways we can guarantee a solution exists—or quickly demonstrate that it doesn't.

Chilis:

Homework: Optional.

Prerequisites: It will help to be familiar with some graph-theoretic terminology.

PARTIAL DIFFERENTIAL EQUATIONS. (Ben and Assaf, Tuesday–Friday)

PAIN IS WEAKNESS LEAVING THE BODY! PARTIAL DIFFERENTIAL EQUATIONS? MORE

LIKE PAINFULLY DIFFICULT EXERCISE! WANT FOURIER TRANSFORMS? WANT INFINITE DIMENSIONAL VECTOR SPACES? WANT INTEGRATION BY PARTS? HAVE YOU EVER TAKEN THE EXPONENT OF A DERIVATIVE? SOLVE ALL OF QUANTUM MECHANICS, SOLVE HEAT AND MOTION AND WAVES AND MAKE LAPLACE'S EQUATION LOOK LIKE ADDITION! YOU WILL TRANSCEND TIME ITSELF. IN THIS CLASS, YOU DO THE WORK, YOU GET RESULTS! CALL US AT 1888-MCSP-PDE FOR YOUR FREE TRIAL TODAY! Limited time offer, some restrictions apply, misuse of this product may cause blindness and hair loss, not available in Alaska or Hawaii.

Chilis:

Homework: Required.

Prerequisites: Topology, multivariable calculus, complex analysis, linear algebra, nonlinear algebra, measure theory, and an undergraduate degree in physics.

You Can't Solve the Quintic. (Eric, Thursday–Friday)
We'll prove that there's no general formula in radicals that works to produce the roots of a degree 5 polynomial. This particular proof due to Arnold is a beautiful combination of combinatorial and topological arguments. The only things you'll need to know going in are what complex numbers, the quadratic formula, and permutations are; but we'll make tons of connections with classes on group theory, fundamental groups, Riemann surfaces and more from the first 4 weeks of camp.

Chilis:

Homework: Recommended.

Prerequisites: Be comfortable with the complex plane, know the quadratic formula, know cycle notation for permutations.

2:10 Classes

Markov Triples and Continued Fractions. (Véronique and Kayla, Wednesday–Friday)
Markov numbers are integers that appear in the solution triples of the Diophantine equation, $x^2+y^2+z^2=3xyz$. A conjecture, unproven for over 100 years, states that a Markov triple $\{x,y,z\}$ is uniquely determined by its maximum. In this class, we prove a statement related to this conjecture to determine an ordering on subsets of the Markov numbers based on their corresponding rational. Indeed, there is a natural map from the rational numbers between zero and one to the Markov numbers. The proof uses the cluster algebra of the torus with one puncture and a resulting reformulation of the conjectures in terms of continued fractions.

Chilis:

Homework: Recommended.

Prerequisites: None, but cluster algebra is an advantage.

Martin's Axiom. (Susan, Wednesday–Friday)
Induction is awesome! You know what else is awesome? Posets! In fact, my one problem with induction is that I only ever get to do it on well-ordered sets. First I prove my result for zero, then for one, then for two, etc. If I'm lucky, maybe I also get to prove my result for infinite ordinals. That's kinda cool. But nowhere near as cool as it could be—I wanna do induction on posets!

Enter Martin's Axiom: an extra-set-theoretic axiom that allows us to do just that. I take the poset of all partially-proved results, and then glue them haphazardly into a giant, glorious Franken-Sign of a proof that gives us an uncountable result.

Wanna see how it works? Come to this class!

Chilis:

Homework: Recommended.
Prerequisites: None.

Procrastination. (Rice, Friday)
If you're like me, you've had the experience of putting off all your homework until the last minute, despite knowing the whole time that pacing yourself would make your life way easier. Come learn about an elegant mathematical model of procrastination, and about how simply realizing that you're going to procrastinate will (usually) cause you to make much wiser decisions.
Chilis: 🌶
Homework: None.
Prerequisites: None.

Sporadic Groups and Where to Find Them. (*Theo Johnson-Freyd*, Wednesday–Friday)
We will survey the finite simple groups, focusing on the exceptional, aka "sporadic," ones. Most groups are "muggles": they live in repeating families. The sporadic groups are sorted into "generations" according to which exceptional object they act on — the magical landscape inhabited by the exceptional beasts. The first generation acts on Golay's error correcting code, and we will spend some time learning to compute in that code. Then we will discuss how a "gauged quantization" procedure convert Golay's code into Leech's lattice, producing the second generation, and then a second "gauged quantization" procedure converts the Leech lattice into Monster Moonshine and the third generation.

Chilis: 🌶🌶🌶
Homework: Optional.
Prerequisites: Know what the following words mean: finite group, (normal) subgroup, alternating group, vector space, complex numbers, arithmetic mod p.

Two Games and a Code. (Mira, Wednesday–Thursday)
Question: What do the following two games have in common?

(1) You think of a number between 0 and 15. I ask you seven yes/no questions about it. You are allowed to tell at most one lie (or, if you prefer, to answer truthfully throughout). At the end, I'll tell you if and when you lied, and then I'll guess your number! (We'll actually play this in class.)
(2) You and six friends are playing a cooperative game. You are each given a black or white hat at random. As usual, each person can see the color of everyone else's hats and has to guess the color of his or her own. Each of you writes your guess — either "BLACK" or "WHITE" — on a piece of paper, without showing it to anyone else. If a player doesn't want to guess, they also have the option of writing "PASS". Then everyone holds up their papers simultaneously. Your team wins if, among you, you have *at least one correct guess and no incorrect guesses.* ("Passes" don't make you lose, but they don't help you win either.) You are allowed to agree on a strategy before the hats are passed out, but no communication is allowed afterwards. What strategy will maximize your probability of winning, and what is the best probability you can achieve? (We'll play this in class if we have time.)

Answer: The same strategy, based on a perfect code!
Come and find out what that means.
Chilis: 🌶🌶
Homework: None.
Prerequisites: None.

Colloquia

Exceptional Mathematics: from Egyptian Fractions to Heterotic Strings. (*Theo Johnson-Freyd*, Tuesday)
Most of the mathematical universe is regular and repeating, but every once in a while there is an exception, and it leads to all sort of interesting and irregular phenomena. I will explain how the exceptional solutions to a simple problem from antiquity — find all integer solutions to $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}>1$ — lead to 20th and 21st century highlights: topological phases of matter, heterotic string theory, and E_8, the most exceptional Lie group.

Visitor Bios

Theo Johnson-Freyd. I'm a mathematician at the Perimeter Institute for Theoretical Physics (Waterloo, Ontario). If you are looking for a research environment with lots of glass, sky bridges, and spaces for collaboration, check us out — we run programs for high school, undergraduate, masters, and PhD students. My research goal is to understand the spaces of possible quantum systems (with such-and-such properties, e.g. supersymmetry or a mass gap); specifically, my goal is to understand the algebraic structures enjoyed by such spaces. This has "practical" applications in quantum computing and material science, but also "theoretical" applications because spaces of quantum systems are often isomorphic to spaces of interest in pure mathematics. For instance, in my favourite recent paper, titled "The Moonshine Anomaly", I used quantum field theoretic ideas to calculate an important topological feature of the "Monster" sporadic finite simple group.