Contents

Apurva's Projects 2
Read Math 2
Assaf's Projects 2
Fix Assaf's Proofs (and Read Differential Topology) 2
Gerrymander the Dorms 2
Zine-ify Mathcamp 2
Ben's Projects 3
Arduino 3
Three-Person Elections 3
Bill's Projects 3
A Tale of Two Generating Functions 3
Counting Equivalence Classes of Permutations 4
Reading Project: Thirty-three Miniatures 4
Eric's Projects 5
Kronecker-Weber by Analogy with Modularity 5
Ring all the Permutations on 6 Bells 5
Eric and Will's Projects 5
Read the AMS "What is. . . ?" Column. 5
Gabrielle's Projects 6
Integer-Valued Polynomials 6
Proofs of Quadratic Reciprocity 6
Gloria and Kevin's Projects 6
Mathematical Crochet and Knitting 6
J-Lo's Projects 7
Kronecker's Jugendtraum 7
Kayla's Projects 7
Enumerating Perfect Matchings 7
Pyramid Partition Using Dimer Shuffling 7
Topological Graph 7
Kevin and Rice's Projects 8
Lattices of Stable Matchings 8
Lizka's Projects 8
Copeland Knots 8
Geometric Sculptures 8
Modeling the Dorms 9
Mark and Bill's Projects 9
Knight's Tours 9
Mark and Gabrielle's Projects 9
Period(s) of Fibonacci 9
Misha's Projects 10
Design a Dominion Expansion 10
Pesto's Projects 10
Castlefall 10
Rice's Projects 10
Money out of Politics. . . and into Charity 10
Shiyue's Projects 11
Counting Skew Semistandard Set-Valued Young Tableaux 11
Singularities of Algebraic Variety of Intersections of Flags 12
Will's Projects 12
Asymptotics of Spanning Trees 12

Apurva's Projects

Read Math. (Apurva)

Description: Read a book/paper about something in the union of algebra, topology, and geometry. I can also suggest things for you to read if you have a topic in mind.

Structure: We will have regularly scheduled discussion meetings.

Expected Input: Self-study

Expected Output: You should take good notes. There could be a poster presentation depending on how far we get.

Difficulty: 🌶🌶🌶🌶

Prerequisites: None.

Assaf's Projects

Fix Assaf's Proofs (and Read Differential Topology). (Assaf)

Description: Assaf likes waving hands and pretending that everything works out as we expect. Assaf is very bad at deep theory and struggles with long words like "differential", "pull-back", and "stereographic projection." In this project, we'll read through some differential topology – at least enough to understand all of the concepts necessary to formalize THE Intuitive Proof of the Hairy Ball Theorem. We will then write out a fully formal proof that takes the hand waves and turns them into math.

Structure: Reading and applying the material

Expected Input: A lot of motivation and about an hour a day

Expected Output: A new (formal) proof of the hairy ball theorem

Difficulty: 🌶🌶🌶🌶

Prerequisites: Multivariable calculus, topology, linear algebra, real analysis (recommended)

Gerrymander the Dorms. (Assaf)

Description: Release your inner gerrymanderer! Imagine the shapes you could draw! Draconic Jackson Pollacks, Picasso masterpieces of districts that will make even the politicians cry. Now imagine doing this in THREE DIMENSIONS! In this project, we will survey campers, draw maps, and use Mathcamp itself as our canvas for political mischief.

Of course, gerrymandering in 3D also means compactness in 3D...

Structure: Three meetings, once a week

Expected Input: 4 hours

Expected Output: A gerrymander of the dorms

Difficulty: 🌶

Prerequisites: Introduction to Gerrymandering would be helpful but is not necessary

Zine-ify Mathcamp. (Assaf)

Description: A zine is a small booklet (see the ones in the lounge) with a bit of information about a specific subject. Pictures, jokes, short stories, and general nonsense is more than welcome in a good zine. This project is to turn Mathcamp classes into zines. There is no limit to the number of campers who can participate in the project!

Structure: Group meetings

Expected Input: 3 hours per zine

Expected Output: ZINES!

Difficulty: 🌶

Prerequisites: Any Mathcamp class

Ben's Projects

Arduino. (Ben)

Description: Arduino boards let people without much experience work with coding and programming in a relatively hands-on way, letting them see the results of their code concretely.

As I have relatively little experience working with Arduino or with coding, this project will be driven largely by your efforts. I will provide whatever assistance I can in debugging and other support of this project.

Examples of projects that can be found on the Arduino website include wifi-controlled devices, calculators, clocks, among many others. This is limited only by your imagination, your assembly of the electronics, and your code!

Structure: Fairly free-form. We can meet to discuss things weekly if necessary, or correspond by email. We will use whatever format will be best for you.

Expected Input: Working fairly independently to code and build some project using Arduino

Expected Output: An Arduino board, programmed to do something interesting

Difficulty: 🌶–🌶🌶🌶🌶

Prerequisites: None.

Three-Person Elections. (Ben)

Description: If two people compete in an election on a one-dimensional issue space, the (unique) Nash equilibrium occurs when they are both at the "median" of the space.

When three people compete, there is no Nash equilibrium in "pure strategies." That is, if all three candidates pick one point to be at, at least one of them will be unhappy with their choice of position! So, to have an equilibrium, at least one candidate will have to randomize what they're going to do.

In this project, we'll study some basic game theory to understand this problem (and why various standard theorems don't apply to it). Then, we'll try to figure out what equilibria to this game look like when the candidates can use some randomization (if there are any such equilibria).

Structure: Reading/discussion

Expected Input: Meeting once a week at minimum

Expected Output: A better understanding of a complicated game, hopefully some fun.

Difficulty: 🌶🌶

Prerequisites: None.

Bill's Projects

A Tale of Two Generating Functions. (Bill)

Description: This project will explore a strange relationship between two well known generating functions. Let $f(x)$ be the generating function $\sum_{n\geq 0}\binom{2n}{n}x^n$ for the central binomial coefficients and $g(x)$ be the generating function $\sum_{n\geq 0} C_n x^n$ for the Catalan numbers (don't worry if you're not familiar with these!). Whereas the formula for $g(x)$ can be derived through a beautiful combinatorial argument, finding a simple and purely combinatorial derivation of $f(x)$ has historically been much more difficult.

The two generating functions have a surprising relationship with one-another, however: if we define $g'(x) = 2xg(x) - 1$, then $f(x)$ and $g'(x)$ are *negative multiplicative inverses* of each other. That is,

$$f(x)\cdot g'(x) = -1.$$

Is this a relationship a coincidence (I doubt it!) or is there some underlying combinatorial explanation? This project will search for such an explanation. Combining such an explanation with the combinatorial derivation for $g(x)$ would then also imply a nice combinatorial derivation for $f(x)$.

This project would begin by (a) learning/discovering known approaches to deriving formulas for $f(x)$ and $g(x)$, and (b) looking for a combinatorial explanation for the relationship that $f(x) \cdot g'(x) = -1$.

Structure: The first part of the project will be spent learning/discovering background material on the generating functions $f(n)$ and $g(n)$. This may include reading part of a book chapter on generating functions, but will mostly consist of working independently (with hints), and discussing ideas in group meetings. The second part of the project will be attempting to prove the relationship between $f(x)$ and $g'(x)$. This will be mostly hands off, but with regular meetings to discuss ideas.

Expected Input: 2-4 hours/week

Expected Output: A poster at the project fair.

Difficulty:

Prerequisites: Some basic familiarity with generating functions would be useful. But it can also be taught as we go if students are very excited about the project.

Counting Equivalence Classes of Permutations. (Bill)

Description: This project will explore open questions in a topic known as *permutation pattern-replacement*. (This also happens to be a research area in which many of the research papers have been written by past Mathcampers!)

The $\{1234, 3412\}$-equivalence (with no adjacency constraints) is an equivalence relation on the permutations S_n of length n. Two permutations are *connected by a pattern replacement* whenever one can be reached from the other by rearranging some increasing subsequence a_1, a_2, a_3, a_4 of length 4 to be in the order a_3, a_4, a_1, a_2. We say that two permutations x and y are *equivalent* if there is a sequence of permutations $x_1, x_2, \ldots, x_k$ beginning with $x_1 = x$ and ending with $x_k = y$, and such that each pair of consecutive permutations x_i, x_{i+1} are connected by a pattern replacement. It is conjectured that the number of *nontrivial equivalence classes* (i.e., equivalence classes of size greater than 1) under the $\{1234, 3412\}$-equivalence is exactly $(n^3 + 6n^2 + 55n + 54)/6$ for $n \geq 7$. (Though this conjecture has only been verified for $n = 7, 8, 9, 10, 11$, meaning it is not at all clear that the conjecture is necessarily true!)

This project would begin by replicating several known results in the area for related equivalence relations and then would attempt to either prove or disprove the conjectured formula for the number of nontrivial equivalence classes under the $\{1234, 3412\}$-equivalence.

Structure: This will be mostly hands off, with regular meetings to background material, ideas, and progress.

Expected Input: 2-4 hours per week

Expected Output: A poster at the project fair.

Difficulty:

Prerequisites: Comfort with permutations and induction. Students may benefit from taking the week 3 Permutation Combinatorics class though.

Reading Project: Thirty-three Miniatures. (Bill)

Description: In this project we'll read some of the best sections of the book, Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra.

This is one of my favorite mathematics books of all time, because (a) almost everything in it is extremely beautiful; (b) the book is very well written; (c) it taught me to love linear algebra; and (d) it's free! (See `https://kam.mff.cuni.cz/~matousek/stml-53-matousek-1.pdf`)

Structure: We will read portions of the book in small groups (or individually), and then we'll discuss the material together.

Expected Input: 2-5 hours per week (up to the campers).
Expected Output:

Difficulty:

Prerequisites: Familiarity with the following terms: vector spaces, vector space basis, dimension, linear independence, matrix multiplication, and matrix determinant. If students are familiar with eigenstuff, then there are also additional sections of the book that we may read.

Eric's Projects

Kronecker-Weber by Analogy with Modularity. (Eric)

Description: We'll work through a proof of the Kronecker-Weber theorem that any abelian extension of $\mathbb{Q}$ is contained in a cyclotomic extension $\mathbb{Q}(\zeta_n)$ for a root of unity ζ_n. With the given prerequisites, there is one straightforward proof using ramification and assuming the local Kronecker-Weber theorem. We'll talk through that and then formulate another proof that proceeds along the same argument as Wiles' proof for modularity of elliptic curves.

Structure: Weekly meetings and independent reading
Expected Input: 3 hours/week of reading, working on computations, and meeting to discuss the problem

Expected Output: Poster fair poster

Difficulty:

Prerequisites: Algebraic number theory, Galois theory (we'll only use it for extensions with abelian Galois group)

Ring all the Permutations on 6 Bells. (Eric)

Description: In small groups, students will learn some of the basics of change ringing and work up to attempting to ring an extent on 6 bells; this is to perform all 720 permutations on 6 bells in one sitting without repetition. Students should sign up in groups of 2 or 3; I can handle two such groups.

Structure: Evening practices

Expected Input: Evening practices 2 to 3 hours per week.

Expected Output: Ring an extent; if this goes very well we can try for a quarter peal (one quarter of all the permutations on 7 bells). This group of students will likely be prepared to perform something short at the talent show also.

Difficulty:

Prerequisites: None.

Eric and Will's Projects

Read the AMS "What is. . . ?" Column. (Eric and Will)

Description: The AMS publishes a series of short (1 to 2 page) expository articles where an expert gives a quick introduction to a particular mathematical object. We'll try to read one a week as a way to broaden our mathematical horizons. Articles can be found at `http://arminstraub.com/math/what-is-column`; currently there are over 150 available covering topics from across mathematics. I found reading these articles super helpful when I was thinking about what branch of mathematics I wanted to go into.

Structure: The group of campers participating will look through the list and select some articles that seem intriguing, we'll read one a week and have a 1 hour discussion

Expected Input: Read a 1-2 page article each week and meet for 1 for a group discussion of it

Expected Output: Poster fair poster and/or evening academics lounge talks

Difficulty:

Prerequisites: The articles are aimed at a general mathematical audience; we'll try to pick articles that assume less background from the reader

Gabrielle's Projects

Integer-Valued Polynomials. (Gabrielle)

Description: The ring of integer-valued polynomials, denoted $\mathrm{Int}(\mathbb{Z})$, is the ring of polynomials f with rational coefficients such that $f(\mathbb{Z}) \subseteq \mathbb{Z}$. While $\mathrm{Int}(\mathbb{Z})$ is sandwiched between two rings we understand very well, $\mathbb{Z}[X]$ and $\mathbb{Q}[X]$, it is actually very strange. The purpose of this project would be to learn more about this ring, either by reading a few papers or some chapters of a book about it, depending on what the camper(s) is/are interested in/what might be most accessible.

Note: I first encountered this ring as part of a research project I did as an undergraduate, when I had very little number theory or abstract algebra knowledge. A camper with little ring theory knowledge will still be able to make sense of the papers and understand some results about nonunique factorization. A camper with more abstract algebra background can possibly get more out of it. It is up to you!

Structure: Meeting for a few hours every week, as desired, to discuss the papers.

Expected Input: 3-4 hours a week (reading + meeting/discussion)

Expected Output: A poster at the poster fair!

Difficulty: 🌶🌶🌶

Prerequisites: Some ring theory would be helpful but not necessary. Some number theory would be very helpful but not necessary.

Proofs of Quadratic Reciprocity. (Gabrielle)

Description: There are 246 proofs of quadratic reciprocity which draw from many areas of mathematics. The purpose of this project is to read through as many of them as are accessible to the student(s), maybe learn some new mathematics along the way, and determine what insights different proofs can provide.

Structure: Ideally, if we can get a group of students with varied areas of mathematical interest, we could do a weekly group discussion of proofs from very different parts of mathematics.

Expected Input: 3-4 hours a week

Expected Output: A poster at the poster fair

Difficulty: 🌶🌶

Prerequisites: Enough number theory to understand the statement of quadratic reciprocity.

Gloria and Kevin's Projects

Mathematical Crochet and Knitting. (Gloria and Kevin)

Description: One of the best ways to visualize surfaces in three dimensions is to hold them in your hands and play with them. In this project, we'll make our own hyperbolic planes, M obius strips, Klein bottles, Seifert surfaces, Lorenz manifolds and more, all out of yarn or felt or fabric! No previous crocheting, knitting, or sewing experience is necessary.

Structure: We'll have introductory "how to" sessions; after that, you can work on projects pretty much wherever and whenever you feel like it! Extra instruction will be needed for Kat Bordhi's M obius cast on, and for grafting. Technical help will be available during TAU.

Expected Input: Totally up to you. For simple things, less than an hour of work is enough. Otherwise, expect a few hours to learn the basics if you've never done any of these things before; after that, it's up to you.

Expected Output: Mathematical surfaces that you can hold and play with!

Difficulty: 🌶

Prerequisites: None.

J-Lo's Projects

Kronecker's Jugendtraum. (J-Lo)

Description: Using a couple of books by Joe Silverman, we will learn about elliptic curves with complex multiplication, with the ultimate goal of understanding how this theory can be used to generate abelian field extensions of imaginary quadratic fields.

Structure: Reading course

Expected Input: Up to participants — suggested 3 hours reading/problem-solving, 30 minutes meeting per week

Expected Output: A worked-through computation of the Hilbert class field of some imaginary quadratic field (possibly in poster form)

Difficulty: 🌶🌶🌶🌶

Prerequisites: Galois theory (how can you find all the subfields of a number field?), algebraic number theory (what can happen to a prime ideal after a field extension?), basic theory of elliptic curves (what is the group law, and what is the group structure?)

Kayla's Projects

Enumerating Perfect Matchings. (Kayla)

Description: I'm interested in running a reading project on different enumeration techniques to count the number of perfect matchings on certain graphs. Specifically, we will be looking at certain tilings of the plane, finding families of finite subgraphs of the tiling, and counting the number of perfect matchings for these. I would like to talk about Kuo condensation, domino shuffling, and potentially one or two other techniques by reading some papers with campers.

Structure: Independent reading / weekly meetings

Expected Input: 1-3 hours per week of reading and 1 hour of meeting weekly

Expected Output:

Difficulty: 🌶🌶🌶

Prerequisites: Having some graph theory exposure, especially to graph matchings, would be nice

Pyramid Partition Using Dimer Shuffling. (Kayla)

Description: I would like to reread a paper by Benjamin Young titled "Computing a Pyramid Partition Generating Function with Dimer Shuffling" [`https://arxiv.org/pdf/0709.3079.pdf`]. It defines a generating function for pyramid partitions via a dimer shuffling algorithm.

Structure: Independent reading + weekly meeting

Expected Input: 2-3 hours a week of reading + 1 hour weekly meeting

Expected Output:

Difficulty: 🌶🌶🌶🌶

Prerequisites: Some exposure to generating functions

Topological Graph. (Kayla)

Description: In this project, we will explore graph embeddings on certain topological surfaces and various properties about them!

Structure: 1 hour a week of reading + 30 minute meeting with me

Expected Input:

Expected Output:

Difficulty: 🌶

Prerequisites: None.

Kevin and Rice's Projects

Lattices of Stable Matchings. (Kevin and Rice)

Description: Suppose we have n bears and n campers. Each bear has a preference ranking over campers and each camper has a preference ranking over bears. We would like to create a one-to-one matching between the bears and the campers. One desirable property of matchings is *stability.* A matching M is called *stable* if there is no bear-camper pair (b, c) such that b and c prefer each other over their match in M. The reason stable matchings are desirable is that in an unstable matching, there is a bear and a camper who would want to jointly defect and leave the matching.

The "fundamental theorem" of the study of stable matchings is that no matter what preferences the bears and campers have, a stable matching always exists. In some cases only one stable matching exists, but usually there are many stable matchings. It turns out that the stable matchings for a given preference profile form a very nice combinatorial structure called a lattice, a very special kind of poset. Roughly speaking, the partial order is that $M \geq M'$ if M is preferred to M' by the bears (and, although this isn't obvious, it follows that M' is preferred to M by the campers).

This project will start with a lecture by Rice about stable matchings and a lecture by Kevin about lattices. You will then work in a small group to investigate interesting questions about the lattice of stable matchings. One example of a question you might investigate is: there is a famous algorithm that generates the minimal matching and the maximal matching of the lattice. But you may want to create a matching that isn't biased toward bears or toward campers. Can you design an algorithm that finds a matching that is somewhere near the middle of the lattice?

In this project, Kevin and Rice will suggest some questions to work on, but you'll basically have free rein to investigate whichever questions you find interesting!

Structure: After two teaching sessions during TAU, students will do independent research in a small group and check in with us on occasion.

Expected Input: After the teaching sessions, however much you want! At least four hours of independent work in your group is recommended.

Expected Output: Whatever you want! (Likely a project fair presentation poster.)

Difficulty: 🌶🌶

Prerequisites: None (we'll teach you about stable matchings and lattices).

Lizka's Projects

Copeland Knots. (Lizka)

Description: Tim! keeps a ball of twine in his pocket as he walks around the campus. When he leaves his room in the morning, he ties one end of the twine ball to the door of his room. He then lets the twine unspool throughout the day, and whenever he crosses the twine on the ground, he steps over it. What kind of knots can he make this way? Is it possible to make all possible knots this way? This project could easily partner with the Modeling the dorms project.

Structure: Meetings with me but mostly independent

Expected Input: 5-7 hours per week
Expected Output:

Prerequisites: Have some familiarity with knot theory (for instance, know the Reidemester moves)

Geometric Sculptures. (Lizka)

Description: Math takes many exciting forms – let's take some and make them in 3 dimensions! What you do: pick a form or structure you especially enjoy and brainstorm with me about some good ways to model it (will it be a complicated cardboard construction? a fabric-and-wire masterpiece? who knows?), then build it!
Structure:

Expected Input: Around 2 hours per week (or more if you get excited)

Expected Output: One or more mathematical models

Prerequisites: None.

Modeling the Dorms. (Lizka)

Description: Our dorms are quite labyrinthine—let's try to make a model of them! This project will be heavily influenced by your preferences. One idea might be to gather info about the dorms and try to make a nearly-to-scale cardboard model. Another might be to make some sequence of variations on them (maybe you have better ideas for our dorms' architecture, or maybe you'd just like to see how many floors you can add to the C tower while preserving its relationship with D tower). This can be a group project, and can also partner up with the Copeland Knots project.

Structure: Pretty independent. Meet with Lizka to discuss plans and continue meeting to check in.

Expected Input: Around 3 hours per week, or more depending on how it goes

Expected Output: A model of our dorms

Difficulty: 

Prerequisites: Have experience with Copeland

Mark and Bill's Projects

Knight's Tours. (Mark and Bill)

Description: Suppose you have a rectangular "chessboard", say of dimensions m and n, and you have a knight that can make standard moves on this board. (A knight's move is a jump that combines a displacement of two squares in one direction – up, down, left, or right – with a displacement of one square in a perpendicular direction.) For what values of m and n can the knight "tour" the board, that is, start somewhere and move to all the other squares in some sequence, without ever visiting the same square twice? And what if we also ask that the ending square is one knight's move away from the starting square, so that the knight's tour can be "closed" by making one more move and returning to the starting square? For our project, you will be investigating this problem (or pair of problems), and perhaps even solving it completely.

Structure: Largely hands-off; we'll be happy to talk about the problem and we hope you'll check in once in a while, but the more the group (of probably 2-3 campers) can discover and prove on its own, the better.

Expected Input: At minimum, several hours of work each week.

Expected Output: A poster at the project fair.

Difficulty: 

Prerequisites: Comfort with proof by induction; patience and willingness to pay attention to detail.

Mark and Gabrielle's Projects

Period(s) of Fibonacci. (Mark and Gabrielle)

Description: Consider the Fibonacci numbers modulo 7:

$$1, 1, 2, 3, 5, 1, 6, 0, 6, 6, 5, 4, 2, 6, 1, 0, \ldots$$

Note that the sequence now starts over, so it is periodic with period 16. What happens if we replace the modulus 7 by some other positive integer n? In this project, you will investigate that. There are several patterns that you can find experimentally; for some of them, you can use basic number theory to show that they will persist, while for at least one of the others, whether the pattern persists is a somewhat notorious open problem. The group (of 3 or so campers) will probably do a bit of (relatively straightforward) programming to get a table of values for the period as a function of n, but after that, you'll be working by hand to understand your findings and/or prove that they will persist.

Structure: Relatively hands-off on the part of the advisors; we'll be available for questions and discussion and we'd like you to check in once in a while, but the more you can discover on your own, the better!

Expected Input: At minimum, several hours of work per week.

Expected Output: At minimum, a poster at the project fair.

Difficulty: 🌶🌶🌶

Prerequisites: A bit of number theory (such as the week 1 course)

Misha's Projects

Design a Dominion Expansion. (Misha)

Description: Dominion is a set-building card game that, since its release in 2008, has seen 11 different expansions with more cards.

In this project, you'll design your own expansion for this card game. This includes:

- Picking a new theme for the expansion (ninjas? dinosaurs? mathematicians?)
- Coming up with lots of cards in that theme that have new exciting mechanics or are just fun to play with.
- Doing lots of play-testing to determine how good the new cards are and how they should be rebalanced.
- Ruthlessly cutting cards that don't work.

Structure: We'll start with a planning meeting, then I'll check in 1-2 times a week on your progress (or whenever you feel like you need advice).

Expected Input: Lots of play-testing.

Expected Output: At least a small expansion (12-13 new cards). If you have the time and energy, maybe a large expansion (25 new cards).

Difficulty: 🌶🌶

Prerequisites: Know the rules for Dominion (at least the base game).

Pesto's Projects

Castlefall. (Pesto)

Description: Castlefall is a game in which half the players have one word from a list of 18 and half have another. The players talk until a player announces a guess at two other players who share their word, or a guess at the other team's word.

(1) How should we think about or model the players' talking?
(2) Why "two other players"? What happens to the game with smaller numbers?
(3) As the game is usually played, after a team is guessed, there's a minute in which a player can guess a word instead. Why?
(4) Strategies?

Structure: Occasional meetings at TAU.

Expected Input: At least an hour of thought.

Expected Output: A better understanding of the game? A poster at project fair?

Difficulty: 🌶🌶

Prerequisites: Having played the game castlefall a few times would help.

Rice's Projects

Money out of Politics. . . and into Charity. (Rice)

Description: In the 2016 presidential election, nearly 2 billion dollars were raised and spent by the Clinton and Trump campaigns. This is an amount of money that, if donated to effective charities, could have saved almost a million lives. Instead, the money that was donated to each campaign was "wasted," in the sense that the money was spent adversarially: to a first approximation, a dollar spent by Trump cancels out a dollar spent by Clinton, with the world gaining almost nothing.

What if, instead, a Trump supporter and a Clinton supporter, each of whom wanted to donate $100 to the respective campaigns, paired off and donated their collective $200 to charity? Taking this a bit further, what if there were a centralized platform that took donations to each campaign and sent matching amounts from either side to charity? (For example, if Clinton raised $100 million through the platform and Trump raised $80 million, then $160 million would be donated to charity and $20 million to Clinton.) If half of the donors on either side participated in the platform, the campaigns would still be well-financed but the world's charities would have a lot more money.

In practice, though, there are problems with this simple idea, the biggest one being that the two sides may be unwilling to pair off dollar to dollar: one dollar may be more valuable to one campaign than the other. So what you could do instead is ask each donor participating in the platform, "What is the maximum number of your own dollars that you are willing to have paired against a dollar from the other side?" Then the platform somehow compatibly pairs donors up. But this opens a whole new can of worms involving incentives: are donors incentivized to answer this question honestly, or to lie in an attempt to manipulate the outcome?

I've done a decent amount of thinking about this question, but I don't have a satisfying solution. Now it's your turn to give this problem a try. If you succeed, or even make progress, maybe you'll make the world a better place!

Structure: Ideally this project will be done in a small group (about 3). The structure will mostly be up to you (and your collaborators), though I'm envisioning a structure where you meet with me a couple times a week and try ideas out as a group in between meetings. But this is really up to you, and I'm happy to accommodate different levels of commitment!

Expected Input: This is mostly up to you, though it would be good if you could spend at least a couple hours a week.

Expected Output: Also mostly up to you, but I'm envisioning a summary of findings and a poster for the project fair.

Difficulty:

Prerequisites: Some exposure to calculus would be helpful but isn't strictly necessary.

Shiyue's Projects

Counting Skew Semistandard Set-Valued Young Tableaux. (Shiyue)

Description: Enumerative geometry is a subfield of algebraic geometry that cares about counting the number and/or dimension of intersections of algebraic varieties. Many of the counting problems in algebraic geometry can be captured by Young tableaux. For example, algebraic geometers may want to ask: how many lines pass through 4 generally positioned lines? The answer is 2, and can be rigorously formulated and elegantly answered in terms of symmetric functions, Schur functions and counting a particular type of Young tableaux.

The algebro-geometric origin of the question is the moduli space of all maps of a general curve into projective space of dimension r and of degree d, but with some extra "imposed conditions." Eisenbud and Harris proved that this space is indeed a projective variety. the "imposed conditions" helps us boil the major counting problems down to counting the number of skew-shaped semistandard Young tableaux (which are just stacked boxes with numbers filled in but with a "upper left fringe" and a "lower right fringe").

The exact question we are answering is hence purely combinatorial: Given a skew shape σ and a number $r \geq |\sigma|$, how many set-valued standard Young tableaux of shape σ with r labels are there? More generally, given a skew shape σ and a vector $c = (c_1, c_2, \ldots)$, how many semistandard set-valued tableaux of shape σ and content c are there?

This project involves understanding of the Robinson–Schensted–Knuth algorithm/correspondence (which is explained in detail in Week 3 Young tableaux and Combinatorics class) and a small expansion of the algorithm to skew shaped semistandard Young tableaux. The reduction of an enumerative geometry problem into computing the coefficients of polynomials using Young tableaux belongs to the realm of Schubert calculus (which is explained in Week 4 Young tableaux and Enumerative Geometry).

We shall prove the correspondence and formulate the counts, seeing it in the general context of skew stable Grothendieck polynomials whose bases encode rich combinatorial, representation-theoretic and geometric information.

Structure: 1-2 students working on this
Expected Input:

Expected Output: An expansion of RSK algorithm to standard Young Tableaux to skew shaped semistandard Young tableaux; result of the count of such Young tableaux.

Difficulty:

Prerequisites: Concurrently taking Young tableaux and combinatorics is necessary, but you definitely don't have to have seen Young tableaux before camp.

Singularities of Algebraic Variety of Intersections of Flags. (Shiyue)

Description: Given a vector space V of dimension n, a flag is a chain of subspaces:

$$V_0 \subseteq V_1 \subseteq \cdots \subseteq V_n = V,$$

where each V_i has dimension i. (They do look like flags under 3 dimensions.) Turns out that if you collect all such flags, they form an algebraic variety called the flag variety. A flag variety of a vector space has amazing properties of being a smooth projective variety, and it looks the same everywhere ("homogeneous"). However, if we look at pairs of flags that intersect given a particular rule, prescribed by a symmetric group element, and collect all the pairs. The total collection might not be a smooth geometric object.

This project involves thinking about different types of intersections of flags and how symmetric group elements define different intersection rules on them. We get to understand the singularities of such geometric object in a very concrete way (you can draw pairs of flags where singularities occurs).

This is a fun interplay between symmetric groups, representation theory, and algebraic geometry. What's amazing is that you are doing all these as if you are doing linear algebra, thinking about permutations and just having fun doodling flags.
Structure:

Expected Input:

Expected Output: A clear presentation on the question; solid understanding on where singularities occur on such varieties and what kind of flags these singularities correspond to (doodling suffice); a description of singularities in terms of symmetric group elements.

Difficulty:

Prerequisites: Linear Algebra, Symmetric Groups

Will's Projects

Asymptotics of Spanning Trees. (Will)

Description: The number of spanning trees of a graph is a natural measurement of its connectedness and complexity. A natural question, then, is to find graphs that have a particularly large number of spanning trees relative to how many vertices and edges they have. However, it's not clear what the "relative to..." should mean in this context, since there's no obvious dependence of the former number of the latter.

One way of understanding how the number of spanning trees depends on the basic parameters of the graph is to look at families of graphs defined by single rules – such as complete graphs, or ladder-like graphs formed by chaining many copies of a small graph together – and investigate how the number of spanning trees grows as the graphs do. We can use powerful tools from linear algebra to deduce many things about these asymptotics, but there's still a lot I don't know and want to learn more about.

This project will be more about asking good questions than resolving specific problems. So you might be interested if you want to get your hands dirty computing with graphs and developing conjectures about them.

Structure: I'll give you some introductory relevant formulas on spanning trees to start out with, and expect you to code it in some way (so as to test predictions on big graphs). After that, I'd like to meet with you 2-3 times a week at TAU to talk about ideas you may have tested and suggested new directions to look at.

Expected Input: See "Structure."

Expected Output: Some conjectures about graphs, some code that can test those conjectures, and maybe some answers.

Difficulty: 🌶🌶🌶

Prerequisites: You should be comfortable with the basic ideas of graphs, somewhat comfortable with manipulating matrices, and ideally able to program in some language that is capable of computing with giant matrices (though you don't need to already know how to compute with giant matrices).