CLASS CLUSTERS FOR MATHCAMP 2019

0. About Clusters

The Mathcamp schedule provides a plethora of options, and we give you the freedom to choose among them as you will.[1] However, you may find the academics this summer more rewarding if you're able to look back in five weeks and reflect that you've learned *a lot* about something. To this end, we've constructed *clusters*: sets of classes with a common theme.

It is completely fine to only take part of a cluster. Note that fewer than half of the classes this summer are in a cluster; just because a class doesn't live in a cluster doesn't mean that it's not awesome!

1. Algebraic Number Theory

Sometimes the rational numbers are not enough, but the real or complex numbers are too much. If that's the case, how about the algebraic numbers? These numbers—solutions to polynomial equations with rational coefficients—have a rich algebraic and number-theoretic structure, which you'll get to see from all sides if you take these classes.

Classes in this cluster:

- **Introduction to Algebraic Number Theory** (🐇🐇🐇, J-Lo, Week 2)
- **Units in Algebraic Number Theory** (🐇🐇🐇, Kevin, Week 3)
- **Reciprocity Laws in Algebraic Number Theory** (🐇🐇🐇🐇, Eric, Week 4)
  Prerequisites: Introduction to Number Theory, Introduction to Algebraic Number Theory

2. Algorithms

The study of telling computers how to solve problems began with Euclid over 2000 years ago. Much later, when computers appeared, people began to finally see the point.

Whenever you ask "how do I solve all problems like this one?" instead of "how do I solve this problem?" algorithms are your friend. These classes will give you many different answers to that question. Computers may or may not be involved.

Classes in this cluster:

- **Super Mario Bros. is NP-hard** (🐇🐇, Tim!, Week 1)
- **Not Your Grandparents' Algorithms Class** (🐇🐇, *Sam Gutekunst*, Week 1)
- **Algorithms in Number Theory** (🐇🐇, Misha, Week 2)
  Prerequisites: Number Theory
- **Probabilistic Models and Machine Learning** (🐇🐇🐇, Mira, Week 3)
  Prerequisites: calculus
- **Randomized Algorithms** (🐇🐇, Bill, Week 4)

[1] Unfortunately, we don't have any time turners to hand out.

3. All About Young Tableaux

A Young tableau is a diagram like the one below:

1	3	4	7	8
2	5	9		
6				

This summer, Shiyue will tell you all about these magical combinatorial objects which help us solve problems in algebra and geometry.

Classes in this cluster:

- **Young Tableaux and Representation Theory** (🔥🔥🔥🔥, Shiyue, Week 2)
  Prerequisites: Group Theory, Linear Algebra
- **Young Tableaux and Combinatorics** (🔥🔥, Shiyue, Week 3)
- **Young Tableaux and Enumerative Geometry** (🔥🔥🔥, Shiyue, Week 4)
  Prerequisites: Linear Algebra

4. The Cluster Algebra Cluster

When you draw a picture with arrows in combinatorics, you call it a directed graph. When you draw the same picture in algebra, you call it a quiver, and you use it as a recipe for constucting algebraic objects with new and unique structure.

Classes in this cluster:

- **Cluster Algebras** (🔥🔥🔥, Véronique, Week 1)
- **Quiver Representations** (🔥🔥🔥🔥, Will, Week 3)
  Prerequisites: Linear Algebra
- **Representation Theory of Associative Algebras** (🔥🔥🔥🔥, Véronique, Week 4)
  Prerequisites: Quiver Representations

5. Complex Analysis

When solving problems over the real numbers fills us with pain and suffering, we turn to the complex numbers for relief. In this world, polynomials always have roots, differentiable functions are also friendly in literally every way possible, and numbers that Descartes called "imaginary" explain mysteries hidden both in the real world and in the real numbers.

Classes in this cluster:

- **Functions of a Complex Variable** (🔥🔥🔥, Mark, Weeks 2–3)
  Prerequisites: Multivariable Calculus
- **The Weierstrass $\wp$ Function** (🔥🔥🔥🔥, Assaf, Week 3)
  Prerequisites: Complex Analysis (one week of Functions of a Complex Variable is enough)
- **Riemann Surfaces** (🔥🔥🔥, Apurva, Week 4)
  Prerequisites: Complex Analysis (one week of Functions of a Complex Variable is enough)

6. Graph Theory

Graph theory is the branch of math that ties together questions about coloring maps, drawing shapes without lifting your pencil, playing "Six Degrees of Wikipedia", analyzing the structure of social networks, and more.

Whenever you have a set of objects with connections between them, whether it's a molecule or a Facebook, you have a graph. These classes will teach you some cool things you can do with them.

Classes in this cluster:

- **Infinite Graphs** (🏃🏃🏃, *Mia Smith*, Week 1)
- **Hedetniemi's Conjecture** (🏃🏃, *Yuval Wigderson*, Week 2)
- **Graph Coloring and Containment** (🏃🏃, Pesto, Week 3)

7. Math and the Real World

One of the most surprising aspects of mathematics is how useful even the most abstract theory can be in applications. Classes in this cluster will teach you some of the ways that we use math to understand, appreciate, and make decisions about the real world.

Classes in this cluster:

- **Introduction to Gerrymandering** (🏃, Assaf, Week 1)
- **Mathcamplandia** (🏃, *Luke Joyner*, Week 2)
- **Thinking of Images as Mathematical Objects** (🏃, *Olivia Walch*, Week 3)
- **The Mathematics of Fairness** (🏃, Mira, Week 4)

8. The Power of Groups

These classes will show you how group theory can be used to solve all sorts of problems outside it. Lots of objects in mathematics and in the real world have group structures, and being able to exploit these is the power of group theory.

Classes in this cluster:

- **Change Ringing** (🏃🏃, Eric + Tim!, Week 1)
- **Group Theory & Rubik's Cubes** (🏃🏃🏃, Gabrielle, Week 2)
  Prerequisites: Group Theory
- **Breaking Bad (RSA Encryption)** (🏃🏃🏃, Michael, Week 3)
- **Musical Lattices** (🏃🏃, J-Lo, Week 4)

9. Problem Solving

Classes in this cluster will teach you problem-solving techniques for problems both in math competitions and beyond. Expect to solve many math problems along the way!

Classes in this cluster:

- **Problem-Solving Cornucopia** (🏃—🏃🏃🏃🏃, Mark, Week 1)
- **Multi-Coefficient Solving of Polynomials** (🏃🏃🏃🏃, Pesto, Week 2)
  Prerequisites: Linear Algebra
- **Problem Solving: Induction** (🏃🏃, Misha, Week 3)

10. Topology

As the old joke goes, a topologist is a mathematician who cannot tell a donut from a coffee cup. The classes in this cluster will introduce you to a more flexible way of thinking about the shapes of various objects.

Classes in this cluster:

- **Homological Algebra** (🐌🐌🐌🐌, *Jeff Hicks*, Week 1)
- **Topology** (🐌🐌, Kayla, Week 2)
- **Fundamental Groups** (🐌🐌🐌, Kayla, Week 3)
  Prerequisites: Group Theory, Topology
- **The Hopf–Poincaré Index Formula** (🐌🐌🐌, Assaf, Week 4)