CLASS DESCRIPTIONS—WEEK 5, MATHCAMP 2026

Contents

9:10 Classes

De Rham Cohomology: Detecting Holes with Differential Forms (, Laithy, $\boxed{\text{TW}\Theta\text{F}}$)
How can calculus detect the holes in a manifold? The key turns out to be the following question: given a closed k-form ω, meaning that $d\omega = 0$, does there exist a $(k-1)$-form η such that $\omega = d\eta$? When such an η exists, we say that ω is *exact*. Closed forms are always locally exact, but they need not be globally exact: a hole in the manifold may prevent a local solution from extending to a global one. De Rham cohomology measures precisely this failure.

The exterior derivative organizes differential forms into the *de Rham complex*

$$0 \longrightarrow \Omega^0(M) \xrightarrow{d} \Omega^1(M) \xrightarrow{d} \Omega^2(M) \xrightarrow{d} \cdots \xrightarrow{d} \Omega^n(M) \longrightarrow 0.$$

Because $d^2 = 0$, every exact form is closed. We therefore define the k^{th} *de Rham cohomology group* by

$$H^k_{\text{dR}}(M) = \frac{\ker\big(d : \Omega^k(M) \to \Omega^{k+1}(M)\big)}{\operatorname{im}(d : \Omega^{k-1}(M) \to \Omega^k(M))} = \frac{\{\text{closed } k\text{-forms}\}}{\{\text{exact } k\text{-forms}\}}$$

These cohomology groups are real vector spaces, and together they form a graded-commutative algebra

$$H^*_{\text{dR}}(M) = \bigoplus_{k=0}^{n} H^k_{\text{dR}}(M)$$

under the wedge product.

In this class, we will prove the Poincaré lemma and homotopy invariance, encounter the gloriously intimidating Mayer–Vietoris sequence, and compute the de Rham cohomology of circles, punctured Euclidean spaces, spheres, and the torus. We will see how these algebraic objects distinguish spaces with different kinds of holes and, if time permits, glimpse at the de Rham theorem: differential forms detect all of a manifold's real cohomological information.

Class format: Interactive lectures

Prerequisites: The differential forms class.

Homework: Recommended

Make Your Own Fractal (, Ben, $\boxed{\text{TW}\Theta\text{F}}$)
Fractal constructions are often created by very rigid self-similarity: carve a square into 9 parts, remove the middle part, repeat, fractal. Break a cube into 27, cut out 7 of those, repeat, fractal. Add this

series of sines and cosines in a certain way, chop up the interval into the Cantor set, fractal fractal fractal.

Obviously, nature is not doing anything so rigid, and yet *natural constructions are also often described as fractals*. This suggests that we might want a less rigid way of constructing fractals, as a "first step" towards generating things that look somewhat natural.

Here, we'll learn about **iterated function systems** on metric spaces, which are one fun way to create a whole bunch of fractals!

Class format: Lecture, with some pictures

Prerequisites: It will help to know what a metric space is, but we'll quickly review that at the start of class.

Homework: Recommended

Probabilistic Gems (🌶🌶🌶, *Milan Haiman*, TWΘF)
We will start by introducing the probabilistic method with some classical examples. Then, we will see several creative examples that I call "gems". These use randomness in surprising and unexpected ways to solve many different kinds of problems.

Class format: Interactive lecture

Prerequisites: Some familiarity with the basics of probability (for example: expectations, independence, and conditioning) will be helpful, but not strictly required.

Homework: Optional

Quadratic Reciprocity (🌶🌶, Mark, TWΘF)
Let p and q be distinct primes. What, if anything, is the relation between the answers to the following two questions?

(1) "Is q a square modulo p?"
(2) "Is p a square modulo q?"

In this class you'll find out; the relation is an important and surprising result which took Gauss a year to prove, and for which he eventually gave six different proofs. You'll get to see one particularly nice proof, part of which is due to one of Gauss's best students, Eisenstein. And next time someone asks you whether 101 is a square modulo 9973, you'll be able to answer a lot more quickly, whether or not you use technology!

Class format: Interactive lecture

Prerequisites: Some basic number theory (if you know Fermat's Little Theorem, you should be OK)

Homework: Optional

Problem Solving (🌶🌶 → 🌶🌶🌶🌶, Mark, TWΘF)
This is not a class that systematically covers problem-solving techniques. Instead, you'll get a variety of attractive (I hope) problems, and just like in "real life", part of the challenge of each problem will be deciding how to approach it and what technique(s) might be relevant. The chili level varies by problem, usually from 2 to 4 chilis. The "optional" listing for homework simply means that you are, of course, welcome to keep working on any problems you don't finish during the class period.

Class format: Mostly work by campers, in groups or individually as they wish; I circulate, offer hints when desired, and may occasionally lecture on something for a few minutes.

Prerequisites: Vary by problem, but most often either "none" or "some calculus". If a problem has a prerequisite that isn't obvious (for example, the problem doesn't contain an integral sign, but integration is needed to solve it), I intend to label the problem accordingly.

Homework: Optional

You CHOICE (, Narmada, $\boxed{\text{TW}}\Theta\text{F}$)

Are you tired of being constrained by your ability to only make finitely many choices at a time? Do you wish that you could write '$x_1, x_2, \ldots,$' where $\ldots$ represents an uncountable infinity? Good news! In this class, YOU CHOOSE!

In this class, we'll talk about what the Axiom of Choice actually says, why it's obviously true, and why it's obviously false. At the end of the class, YOU CHOOSE whether you accept the Axiom of Choice, reject the Axiom of Choice, or a secret third option.

Class format: Interactive lecture

Prerequisites: 8 hours of sleep.

Homework: 8 hours of sleep

Fourier Analysis but Actually Quantum Computing (, Glenn, $\text{TW}\boxed{\Theta\text{F}}$)
Shor's algorithm allows us to factor large numbers in polynomial time on a quantum computer, something that most people don't think is possible on classical computers. Surprisingly, this number theory problem is solved on quantum computers using Fourier transforms! We'll introduce the quantum computing model from scratch and show how Shor's algorithm works, connecting these seemingly distinct fields.

Class format: Lecture

Prerequisites: Fourier Analysis but Actually Algebra. Talk to Glenn if you have other Fourier background, e.g. from Bowen's or Ben's classes; that might work too.

Homework: Recommended

10:10 Classes

Claude Writes a Mathcamp Class: Sperner's Lemma and the Brouwer Fixed Point Theorem (, Claude, $\boxed{\text{T}}\text{W}\Theta\text{F}$)
Note from Della: I asked the free version of Claude (Sonnet 4.6) to write a class from scratch. It chose the topic, chili level, etc. I've checked that it looks superficially like a math class, but haven't checked it carefully—we'll find out whether it's correct or any good when I present the script Claude wrote for me. And then we'll discuss what worked, what didn't work, and how actually teaching a Mathcamp class can't be AI-generated. What follows is the blurb Claude wrote.

Crumple a map of Vermont into a ball, then flatten it back out and lay it on top of a fresh, uncrumpled copy of the same map—with the borders lined up. Somewhere on the crumpled map is a point resting *exactly above* the real geographic location it represents. This is an illustration of the **Brouwer Fixed Point Theorem**: any continuous function from a closed disk (or triangle, or any "blob without holes") to itself must leave at least one point unmoved. The theorem sounds like it belongs in a graduate topology course. In this class we will prove it in 50 minutes, using nothing beyond a counting argument about colored triangles.

The key tool is **Sperner's Lemma**. Take a big triangle, subdivide it into smaller triangles (a *triangulation*), and color every vertex red, green, or blue. The only rule: along each edge of the big triangle, you may only use the two colors that appear at that edge's endpoints—no sneaking in a third color on the boundary. Interior vertices may be colored however you like. Sperner's Lemma says that no matter what you do, the number of small triangles with all three colors—one vertex each of red, green, and blue—is always *odd*. The proof, which you will carry out in class, fits in a paragraph and turns on a parity argument. The surprise is what comes next: applied to cleverer and cleverer triangulations, this combinatorial fact about colorings forces a fixed point into existence. A tiny miracle produces a big theorem.

Class format: Interactive lecture with a short group-work activity (try your own Sperner coloring).

Prerequisites: Comfort with proofs, especially parity / mod 2 arguments. No topology, calculus, or analysis required.

Homework: Recommended

Proving Games Hard (🌶🌶, Della, TWΘF)

There have been multiple classes on NP-hardness this summer. But did you know there are more complexity classes? I'll tell you about another one that appears a lot, especially in 1-player or 2-player games: PSPACE. We'll see some PSPACE-complete problems, sketch the proof that PSPACE = NPSPACE, and develop some tools for proving different types of games PSPACE-hard.

Class format: Lecture

Prerequisites: My *Proving Puzzles Hard* class, Dan's *The Limits of Computation*, or similar

Homework: Recommended

A "Topological" Proof of the Infinitude of Primes (🌶🌶🌶, Purple, TWΘF)

Possibly the most famous proof in mathematics is Euclid's proof of the infinitude of primes. This is a beautiful arithmetic argument. In this class, we will discuss an alternate topological proof due to Furstenberg, which some people consider to be quite beautiful in its own right. Then we will meditate together on whether this proof really counts as topological.

Class format: Lecture

Prerequisites: point-set topology. At minimum, you should be able to prove without much effort that:

(1) the finite union of closed sets in a topological space is closed, and
(2) if B is a basis for a topology T and B_1, B_2 are elements of B with x in their intersection, then there is a basic neighborhood B_3 of x which is contained in that intersection.

How to Teach Fractions (🌶, Riley, TWΘF)

Fractions are a big deal for Riley. He is scared of teaching them. If you want to help him become less scared of teaching fractions, come to this class and share your thoughts on how to teach fractions.

This class is a discussion-based class on how to teach fractions at the late elementary and middle school level. Fractions are important in math teaching because they are probably the first significant mathematical abstraction children face in their mathematical lives. You will be provided readings about how to teach fractions from a few different perspectives and will discuss them in class.

Class format: Small amount of lecture, largely discussion

Prerequisites: None.

Homework: Optional

Can Every Crossing Be Untangled? (🌶🌶🌶, Bowen, TWΘF)

A planar graph is a graph that can be drawn in the plane without any edge crossings. In particular, in such a drawing, every pair of edges crosses an even number of times—namely, zero times.

Crossing parity is surprisingly robust. Many local changes to a drawing may create or remove crossings, but they change the number of crossings between two edges by an even number. This suggests a natural reverse question: Suppose a graph can be drawn so that every pair of edges crosses an even number of times. Must the graph be planar?

The Hanani–Tutte theorem gives a remarkable positive answer: even crossings can be untangled.

In this one-day class, we will explore why crossing parity can detect planarity and discuss two proofs of the theorem. One proof is elegant and topological, using surfaces and closed curves. The other is more direct and intuitive, based on untangling the drawing one edge at a time.

Class format: Lecture

Prerequisites: You should know what vertices and edges are. No prior topology or advanced graph theory is required.

Homework: Recommended

The Finite Field Kakeya Problem (🌶🌶🌶, Charloutte, $\boxed{\text{TW}}\Theta\text{F}$)
The *Kakeya conjecture* is a major open problem in harmonic analysis - it concerns the size of a *Kakeya set*, which is a set in $\mathbb{R}^n$ that contains a unit line segment in every direction. If you went to Alan's colloquium, you'll have heard that these sets can be very small - they can have zero area/volume! - but the conjecture is that they must be large in the sense of dimension. If you didn't go to Alan's colloquium, that's OK, because we're not going to work in $\mathbb{R}^n$ at all, and we're not going to do much analysis either (don't tell Ben).

Instead, we'll look at the finite field version of this problem, which was solved in 2010 by Dvir using the *polynomial method*. The proof is surprisingly elegant and simple (relatively, at least), and inspired many other mathematicians to tackle many other problems in combinatorics and analysis using this method. We won't have time for the other applications, but we will see Dvir's original proof.

Class format: Interactive lecture.

Prerequisites: You should be happy to work in the world of $\mathbb{Z}/p\mathbb{Z}$, the ring of integers modulo a prime p, and recognize this as a finite field. You should also have some linear algebra background, including feeling comfortable with the Rank-Nullity Theorem.

Homework: Recommended

Integer Points in Polytopes (🌶🌶🌶, Narmada, $\text{TW}\boxed{\Theta\text{F}}$)
There's an ancient saying that goes, "What a mathematician can do in two years, a Mathcamp class can do in two days." This class is based on a research problem I worked on in my first two years as a graduate student: if I fix some parameters of a lattice polytope in $\mathbb{R}^n$, can I say something about how many integer points it has? This problem is well-studied for fixed n, but what about arbitratily large n? It turns out that all of the classical counting arguments involve horrible constants in terms of n. In this class, I'll convince you why this problem is interesting, and talk about two recent papers that (almost) entirely solve the problem.

Class format: Interactive lecture.

Prerequisites: Comfort with basic discrete probability (sums of random variables and Markov's inequality); Know some linear algebra (linear independence, matrix multiplication); Believe in multivariate polynomials.

Homework: None

A Graph Walks into a Shredder (🌶🌶🌶, Ce, $\boxed{\text{TW}}\Theta\text{F}$)
A graph has been fed into a very peculiar shredder. Instead of destroying it completely, the machine produces one card for each vertex, showing what remains after that vertex is deleted. The cards are shuffled, and the original graph disappears. Can we put it back together? We will play detective with these "decks", recover surprisingly much information from them, and reconstruct several families of graphs. Then we will meet the central mystery: does the deck always determine the original graph? Despite decades of effort, no one knows.

Class format: Interactive lecture.

Prerequisites: None.

Homework: None

Polynomials That Refuse to Vanish (🌶️🌶️🌶️🌶️, Ce, TW$\boxed{\Theta\text{F}}$)
What can one coefficient of a polynomial possibly tell us about a combinatorial problem? Surprisingly, quite a lot. The Combinatorial Nullstellensatz says that if a polynomial contains the right nonzero coefficient, then it cannot vanish everywhere on a sufficiently large grid—even if finding an explicit nonzero point seems impossible. We will develop this theorem from polynomial interpolation and learn how to recognize the coefficient that matters. Then we will use it to prove the Cauchy–Davenport theorem: adding two sets of residues modulo a prime forces the resulting sumset to grow. Depending on time, we may explore restricted sumsets, repeated addition, or another application in which a polynomial proves that a desired combinatorial object must exist.

Class format: Lecture

Prerequisites: Comfort with polynomials and modular arithmetic; familiarity with finite fields.

Homework: None

The Magic of Determinants (🌶️🌶️ → 🌶️🌶️🌶️, Mark, $\boxed{\text{TW}\Theta}$F)
Even if you have taken some linear algebra, you may not have seen much about determinants in general (beyond the 2×2 and 3×3 cases). If that has left you feeling dissatisfied, either about not really having a good definition (by the way, using the Laplace expansion, while it is computationally sometimes handy, leads to a miserable definition with no intuitive basis at all) or about not having seen many of the properties that determinants have, this may be a good class for you. If all goes well, we'll give a definition of determinant that's both motivated and rigorous, and there will be proofs of all its main properties (such as Laplace expansion), as well as a few applications such as general formulas for the inverse of a matrix and for the solution of n linear equations in n unknowns ("Cramer's Rule").

Class format: Interactive lecture

Prerequisites: Some linear algebra, including linear transformations, matrix multiplication, and determinants of 2×2 matrices.

Homework: Optional

The Putnam (🌶️, Mark, TWΘ$\boxed{\text{F}}$)
You may well have heard of it, but maybe not by its formal name: The William Lowell Putnam Mathematical Competition. As you may know, this is a challenging annual 6-hour exam for undergraduates in the U.S. and Canada; more often than not, the median score is under 10 (out of a possible 120 points). At various times before I retired, I was involved with the Putnam, as a problem setter and later in a more administrative capacity. In this class (if it runs) I intend to talk some about the history and "culture" of the Putnam, tell some stories about it, answer any questions you may have, and try to dissuade you from taking it while you are still in high school (which is possible), with one exception.

Class format: Interactive lecture; Q and A

Prerequisites: None.

Homework: None

11:10 Classes

Mathematics of Common-Sense Postulates (🌶️, *Assaf Bar-Natan*, T$\boxed{\text{W}\Theta\text{F}}$)
Consider the following sensible observations about the world:

- "A relationship is expected to last as long as it's already lasted."

- "Any person is a weighted average of their acquaintances."
- "On average, your friends have more friends than you do."
- "More cheese, more holes. More holes less cheese." (Known as the Swiss-cheese paradox.)

Can you make them mathematically precise? If so, which ones are true? Can you prove them? This class will explore turning our vague beliefs into axioms. Sometimes this goes as planned. Sometimes we find contradictions. Sometimes we end up defining a unique object.

Class format: Interactive lecture

Prerequisites: None, but calculus and basic graph theory can be helpful for some problems.

Homework: Recommended

Outro to Linear Algebra (🌶️🌶️, Glenn, T$\boxed{\text{W}}$ΘF)
In Intro to Linear Algebra, you learned about systems of linear equations. We learned a lot about how to solve them and what structure they have. Instead of equations, this class will take things one step further and wrap things up by talking about systems of linear *inequalities*. Here are some questions that we could try to answer:

- How do you know when a system of linear inequalities has a solution?
- If the system has many solutions, can we optimize for the best solution that minimizes a cost function?
- What are some applications of such systems?

Class format: Interactive lecture

Prerequisites: Nothing is fine, some familiarity with matrices is preferred.

Homework: Recommended

Why the Ruler is Useless (🌶️, Svatava, TW$\boxed{\Theta}$F)
In Euclidean geometry, we often create geometric constructions using a ruler (straightedge) and a compass. In this class, we will show the straightedge is not actually necessary – all of these constructions can be done with only a ruler. This statement (called the Mohr–Masheroni theorem) has a very elegant proof, which will be the subject of this class. Starting with simple constructions, we will expand what we can do with only a compass, ending with "we can do everything".

Class format: Lecture

Prerequisites: None

Homework: Optional

Graphs on Surfaces (🌶️🌶️, Tianlin, TWΘ$\boxed{\text{F}}$)
Which graphs can we draw without crossings on the plane, and how does the answer change when we move to other surfaces? What hidden properties do these graphs, known as planar graphs, satisfy, and why do those properties suddenly change on a torus? In this class, we will discover and derive the remarkable Euler's Formula relating the numbers of vertices, edges, and faces of a graph, use it to solve surprising problems about planar graphs and polyhedra, and then explore how these ideas extend to surfaces with handles. Along the way, we'll see how a single equation connects very different fields such as graph theory, geometry, and topology.

Class format: Interactive Lecture

Prerequisites: You should be familiar with some basic graph theory concepts, namely vertices, edges, cycles, and connected graphs. Familiarity with what convex polyhedra are and knowing a few examples is also recommended.

Homework: Optional

Packing Permutation Patterns (, Misha, T$\boxed{\text{W}\Theta}$F)

Prepare by picking a permutation π and a pattern P. Probabilistically pick $|P|$ pieces of π: perhaps putting the pieces together produces P? Let $\rho_P(\pi)$ be the probability of producing P.

To *pack* P in π is to puff up this probability, making P as plentiful as possible. We will ponder the packing problem for $P = 132$ (and plenty of its pals) using a progression of powerful problem-solving procedures.

Class format: Lecture

Prerequisites: None.

Homework: Recommended

Voting and Chaos (, Ben, TW$\Theta\boxed{\text{F}}$)

It's well-known that *if* voters vote for whichever candidate is closest to them, and *if* everything is on a one-dimensional axis, then both candidates should converge to the median voter—this is called the "Median Voter Theorem;" I've seen it attributed to both Downs and Black.

Only slightly less-well known is this: Never go in against a Sicil—wait, sorry, that's an outdated movie reference. OK, *very* less-well known is this: in a 2-dimensional, or higher-dimensional "issue space," there *is* no optimal strategy[1]. Rather, for *any* position that a candidate could take, there's another position that beats it—and another that beats that, and so on! This part is due to McKelvey.

In fact something weirder is true (also due to McKelvey)—but you'll just have to come to the class to learn about that!

Class format: Lecture, with some picture sketches and sketchy pictures

Prerequisites: Basically none

Homework: Recommended

Thinking Inside the Box: Cube Complexes (, Neelam, T$\boxed{\text{W}\Theta\text{F}}$)

If you glue squares together around a corner, you usually expect them to form a flat plane or the corner of a cube. But what if you keep gluing squares together so that every corner is "thick" or "highly branched," creating a space that branches out like an infinite, multi-dimensional tree made entirely of blocks? Welcome to the world of CAT(0) cube complexes. In this week-long course, we will explore these fascinating spaces from two completely different viewpoints. On one hand, we will look at them geometrically through the lens of hyperplanes: walls that cut through the cubes and divide the space in half. On the other hand, we will look at them combinatorially through a beautiful algebraic object called a pocset (a partially ordered set with an involution). The climax of our week will be mastering Sageev's Construction: a mathematical superpower that allows you to take a purely abstract combinatorial pocset and reconstruct a fully-fledged geometric CAT(0) cube complex out of thin air, and vice versa.

Class format: Lecture and homework recommended

Prerequisites: Point set topology, intro to groups, and graph theory

Homework: Recommended

∞ is Not a Number, but It's a Prime (, August, T$\boxed{\text{W}}\Theta$F)

We will explain why ∞ is a prime. Genuinely. I swear, it really is a prime. Ask anybody.

Some preview of the content: Rational Numbers, Valued Fields, Archimedean and non-Archimedean valued fields, Ostrowski's theorem.

Another on-theme topic is p-adic numbers.

Class format: Lecture

[1] Usually, at least.

Prerequisites: Know what a field is. Know that rational numbers form a field.

Homework: Recommended

Möbius Inversion (🌶🌶, August and Narmada, TW$\boxed{\Theta}$F)
This is a normal class about Möbius Inversion in number theory. August will talk for a while, very normally. Then Narmada will talk for a while, very normally. Nothing strange will happen.

Class format: Lecture

Prerequisites: Introductory number theory (Fermat's Little Theorem).

Homework: None

Solution to the Problem from Mathcamp 2025 Open Problem Board (🌶🌶, Nikita, TW$\Theta\boxed{\mathrm{F}}$)
Last year, Mathcamp had an open problem board, and I posted on it a conjecture of Ivailo Hartarsky.

Take the infinite triangular grid. Every upward-pointing triangle has a lamp, which is either on or off; every downward-pointing triangle has a button, and pressing it toggles the three lamps adjacent to it. One rule: at no moment may more than k lamps be on, for some fixed k. Initially, exactly one lamp is on. The conjecture: you cannot clear an arbitrarily large neighborhood of the start—there is a radius $R = R(k)$ such that no sequence of legal presses ever reaches a configuration with all lamps within distance R of the initial lamp off.

The problem has since been solved, and I'll present the solution.

Class format: Lecture

Prerequisites: None

Homework: None

1:10 Colloquia

Surprise (Napoleon, Tuesday)

What is Randomness? (Bowen, Wednesday)
"The theory of probabilities is at bottom nothing but common sense reduced to calculus," wrote Laplace. But if we pause over this claim, we might ask: What constitutes 'common sense' here? What does it actually mean for something to be random? We will explore both the philosophical and mathematical sides of randomness. How is randomness connected to Hilbert's Sixth Problem, a lesser-known result of Frank Ramsey, and the theory of computation? Come to the colloquium and find out!

Symmetric Groups and Outer Automorphisms (Della, Thursday)
There are $n!$ ways to move n dots around, and they have a natural algebraic structure called the *symmetric group* S_n. We'll explore the question: if all we know is this algebraic structure, how much can we learn about the original n dots? This will mostly involve looking at colorful pictures and pondering numerological coincidences. You'll also find out what's so exciting about the puzzle I've been carrying around since week 2!

Nobody Solves the Quintic (*Dror Bar-Natan*, Friday)
Everybody knows that nobody can solve the quintic. Indeed this insolubility is a well known hard theorem, the high point of a full-semester course on Galois theory, often taken in one's 3rd or 4th year

of university mathematics. I'm not sure why so few know that the same theorem can be proven in about 15 minutes using *very* basic and easily understandable topology, accessible to practically anyone.

2:10 Classes

Arithmetic Ramsey Theory 0 (🌶🌶🌶, Bowen, $\boxed{\mathrm{T}}\mathrm{W\Theta F}$)
Color the integers using finitely many colors. Must one color contain a long arithmetic progression?

Van der Waerden's theorem says yes: no matter how the integers are colored, and no matter how long a progression we ask for, there will always be a monochromatic arithmetic progression of that length.

In this one-day class, we will prove Van der Waerden's theorem using a clever combinatorial argument. The proof shows how increasingly rigid arithmetic patterns can be forced one step at a time, until a monochromatic progression becomes unavoidable.

If you attended the Week 2 class on arithmetic Ramsey theory, this class will give you a new, entirely combinatorial perspective on the subject. If you did not, that is equally fine: the class will be self-contained.

The proof is a striking example of the central philosophy of Ramsey theory: complete disorder is impossible, and sufficiently large structures must contain unexpected patterns.

Class format: Lecture

Prerequisites: Familiarity with mathematical induction and pigeonhole principle.

Homework: Recommended

Napoleon's Favourite Maths (🌶, Napoleon, $\mathrm{T}\boxed{\mathrm{W\Theta}}\mathrm{F}$)
In this class, Napoleon will teach you about all the math that ever existed and will ever exist. In particular, every open problem will be solved, you will attain spiritual enlightenment and your life will never be the same again.

Class format: Waging a war with yourself.

Prerequisites: Knowing how to pronounce "croissant" correctly.

Exploring Extreme x in e^x (🌶🌶🌶, *Assaf Bar-Natan*, $\mathrm{TW\Theta}\boxed{\mathrm{F}}$)
Expo expounds experiments explicitly expanding exponents. The expanded expression expels explosive exploration of $\exp(x)$ and explains its expansive exploits. Explicitly, experience how exponentiation exports expanses to groups and exposes exploitable expressways to solving ODE and PDEs. Before expiring, expect explicit explanations of extreme examples of $\exp(x)$.

In English:

Using calculus, we can write:

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

But what if we took that to be the *definition* of e^x? If that's the case, then we can define e^x for some very weird x's ranging from complex numbers to strange algebra systems to matrices to derivatives... whatever that means. In this class, we will find out what this means!

Class format: Interactive lecture

Prerequisites: None

Homework: Recommended

Hex and Brouwer (🌶🌶🌶, Nikita, $\boxed{\text{TW}\Theta\text{F}}$)
The game of Hex is a game played on the rhombus on the hexagonal lattice. Two opposite sides are colored red and two opposite sides blue. Two players place chips of their color and try to connect the two sides of their color by a chain of chips. It turns out, that there are no draws in this game! This is the Hex theorem. A beautiful result by Gale shows how to derive Brouwer's fixed point theorem from it. He also shows the converse: how to use Brouwer to derive a continuous version of Hex theorem: If points of a rhombus are colored red and blue and red set is closed, then either blue or red sides are in the same connected component of their respective color.

Class format: Interactive lecture

Prerequisites: None.

Homework: Optional

How to Teach (🌶, Ari, $\boxed{\text{TW}\Theta\text{F}}$)
I probably don't need to sell you on the idea that learning from a great teacher is a transformative experience. After all, you're here at Mathcamp. But I might need to sell you on the claim that you—yes, you—can be a great teacher. To my mind, calling somebody a great teacher is like calling them a great pianist. It doesn't mean they made beautiful music the first time they touched the keyboard. It means they've spent countless hours practicing and refining their skills, and gotten feedback and coaching from more experienced pianists, so that what once felt impossible is now second nature.

I have three main goals for this class:

- Provide you with the beginnings of a toolkit for teaching: how to speak, write, interact, and plan for the classroom.
- Invite you to think critically about tough questions: Am I being understood? What's the right mental model for this abstraction? How safe do my students feel speaking up?
- Give you a framework for future improvement: What sort of feedback do you seek out, and from whom? How do you practice and refine your techniques? How do you stay motivated in the face of difficulties?

We'll explore these topics through a variety of class formats. Exactly what we cover will depend on your priorities, feedback, and questions. Let's play some Mozart together! (Or Chopin, if you prefer.)

Class format: Multiple: group discussion, small group discussion, lecture, student presentations

Prerequisites: None

Homework: Recommended

Logic Puzzles (🌶, Misha, $\boxed{\text{TW}\Theta\text{F}}$)
A camper, a mentor, and a JC are crossing a river. The camper has two cabbages and does not trust the mentor. The mentor has two mangos and does not trust the camper. The JC has one very large jalapeño chili and does not trust anyone.

Before they get on the boat, the mentor turns to the JC and says, "I know that you know that I am a vampire."

"I already knew that you were insane," the JC replies.

What color is the camper's hat?

This class is about solving a large variety of puzzles of a similar flavor to the nonsense you just read. There will be some discussion of mathematical theory, but only occasionally. If there are multiple days, they will be independent.

Class format: I will give an introduction to the puzzles at the beginning of class, then give you time to solve puzzles on your own and working together. We will reconvene at the end to discuss some of the interesting things that happen in the solutions.

Prerequisites: None.

Homework: Optional

Lebesgue Ball Knowledge (🌶🌶🌶🌶, Riley, $\boxed{\text{TW}}\Theta\text{F}$)
Who knew that lemmas involving balls show up with the Lebesgue integral? The goal of this class is to cover at least one of the fundamental theorems of calculus for Lebesgue integrals. We will learn about the Vitali covering lemma, the Hardy–Littlewood maximal function, and the Lebesgue differentiation theorem, which is essentially the first fundamental theorem of calculus. If time permits we will continue on to integrating derivatives of functions whose derivatives do not always exist.

Class format: Lecture

Prerequisites: Some kind of real analysis. Know what Lebesgue measure is, or at least believe some believable properties it has.

Homework: Recommended

The One and Only Cantor set (🌶🌶🌶, Maya, $\text{TW}\boxed{\Theta}\text{F}$)
The Cantor set, as a topological space, has many neat properties: it is compact, metrizable, has a basis of clopen sets (i.e., sets which are simultaneously closed and open), and has no isolated points. We will show that it is the only space with these properties. (This is due to Brouwer.)

Class format: Interactive Lecture

Prerequisites: Know what a metric space is, and ideally what a topological space is. Be comfortable with open and closed sets, limits of sequences, completeness, and limit points. Be familiar with compactness (if you're not, just look up the definition).

Homework: Optional

The ZFC Axioms (🌶, Maya, $\text{TW}\Theta\boxed{\text{F}}$)
ZFC, ZFC... you keep hearing these letters. Come to this class to see what the actual axioms are, and see how we get all the standard objects of mathematics from, well, nothing.

Class format: Interactive lecture

Prerequisites: None

Homework: Optional

3:10 Classes

The Condensation of Singularities (🌶🌶🌶, Ben, $\boxed{\text{TW}\Theta}\text{F}$)
This course will unite the topics of *both* of Ben's Week 3 classes—Baire category and Fourier analysis! We will learn a little bit about Banach spaces, and linear maps, and about one of the most important theorems in functional analysis, the Baire Category Theorem, and maybe even another important result, the Baire Category Theorem.

Sorry, my autocorrect got out of control there. That was supposed to say *the Uniform Boundedness Principle* and the *Open Mapping Theorem*. But it turns out that the proof of *both* of those is just "whack things with the Baire Category Theorem," so I often joke that the most important results in functional analysis are the Baire Category Theorem[2], the Axiom of Choice[3], and the Baire Category Theorem[4].

[2] Uniform Boundedness
[3] Hahn-Banach, by the way
[4] Open Mapping

So, what *is* the Uniform Boundedness Principle? How *does* the Baire Category Theorem show up in functional analysis? How is this connected to Fourier series? And what kinds of singularities are we condensing here, anyways?

In this class, we'll answer all of these questions[5], and even see why the Dirichlet kernel is such a bad kernel!

Class format: Lecture

Prerequisites: Have seen some Fourier-related stuff (e.g. Glenn or Ben's W3 class, Laithy and *Alan*'s W4 class). Previous exposure to functional analysis may be helpful, but shouldn't be necessary

Homework: Recommended

The Mathematics of Having Something in Common (🌶🌶🌶, Ce, $\boxed{\text{TW}\Theta}$F)

Suppose we collect k-element subsets of $\{1, \ldots, n\}$, under the rule that every two sets must intersect. How large can our collection be? Taking all sets containing one favorite element is an obvious construction, but is it always the best? We will prove the Erdős–Ko–Rado theorem which answers the question. Then we will start changing the rules. What if no single element is allowed to belong to every set? What if every three sets must have something in common? What if two sets must share several elements rather than just one? We will search for extremal constructions, test tempting conjectures, and see how small changes in the definition can lead to surprisingly different answers.

Class format: Interactive lecture

Prerequisites: Comfort with set notation, binomial coefficients, and counting.

Homework: Optional

More SET Theory (🌶🌶, Della, $\boxed{\text{T}}$WΘF)

I have another SET variant! It's totally different from the ones we saw in week 2—it's about origami instead of group theory. I'll tell you about it, and prove a theorem from my own research about which crease patterns correspond to sets.

Class format: Lecture

Prerequisites: None

Homework: None

The Mathematics of Juggling (🌶🌶, Joy, T$\boxed{\text{W}\Theta}$F)

Will talk about some cool results in the mathematics of juggling, including the Average Theorem and Siteswaps. Will include live demonstrations.

Class format: Lecture

Prerequisites: None

Homework: None

Problem-Solving: Rapid Fire (🌶, Misha, $\boxed{\text{T}}$WΘF)

Not all math competitions are about sitting down to write serious proofs. Sometimes you just have to find the answer as quickly as possible! This class will mostly be an opportunity to practice doing that with some problems I think are fun; it's possible we'll learn about a couple of tricks along the way.

Class format: Timed problem-solving; slides

Prerequisites: None

Homework: None

[5]And maybe not deal with the Open Mapping Theorem—maybe on homework?

How Do You Pick a Random Point on a Sphere? (🌶🌶, Bowen, T$\boxed{\text{W}\Theta}$F)
Suppose you want a computer to choose a point uniformly at random from the surface of a sphere. How hard could that be?

Surprisingly, many obvious methods are secretly biased. Choosing latitude and longitude at random produces too many points near the poles. Choosing random coordinates and scaling the result can preserve traces of the shape you started with. And in higher dimensions, even deciding what 'uniformly at random' should mean becomes less obvious.

In this two-day class, we will investigate a collection of plausible algorithms, test which ones fail, and try to discover what a genuinely random point on a sphere should look like.

Class format: The first day is mainly divided into discussions and activities, while the second day is proof-focused.

Prerequisites: Comfort with vectors and basic probability.

Homework: Required

Truth in Fiction (🌶, Purple, $\boxed{\text{TW}\Theta}$F)
In the Sherlock Holmes universe, Sherlock Holmes lives at 221B Baker Street. In the Sherlock Holmes universe, the Prime Minister is not a pink jellyfish. What gives us confidence in the correctness of these two statements? To answer this question requires developing a theory of truth in fiction; this, in turn, requires understanding modal and paraconsistent logics, among other subjects of interest to analytic philosophers. The goal of this course is to develop such a theory.

Class format: Discussion

Prerequisites: None.

Homework: Recommended