CLASS DESCRIPTIONS—WEEK 4, MATHCAMP 2026

Contents

9:10 Classes

Class Groups (♧♧♧♧, August, TWΘFS)
This is a class about groups that are called class groups.

One of the greatest properties of the natural numbers is unique factorization into primes. However, this fails frequently for other number fields: In $\mathbb{Q}(\sqrt{-5})$, $6 = 2 \times 3 = (1 + \sqrt{-5})(1 - \sqrt{-5})$. Class groups of these number fields measure this failure, and try to salvage the parts we still can.

Some other applications of class groups include:

- If p does not divide the size of the class group of the p-th cyclotomic field, then Fermat's Last Theorem is true for $n = p$.
- The famous prime-generating polynomial.
- Using Class Field Theory, the class group—an "inner" structure of the field—predicts which Abelian extensions—"outer" structures—can exist.

We will attempt to explore the above topics, make explicit calculations of certain class groups, use them to solve Diophantine equations, etc. The main focus will be the class groups of quadratic extensions.

Homework: Recommended

Class format: Lecture

Prerequisites:

- Knowledge of groups (Abelian), including Lagrange's theorem;
- Good command of ring theory, particularly ideals.
- Some modular arithmetic—e.g. understanding that 'half' of the integers mod p are squares mod p.

Fair Division Using Topology (♧♧, *Jane Wang*, TWΘFS)
How can we fairly divide a cake among multiple people when each person values frosting, edges, etc. differently? We can answer this question using tools from topology: the study of continuous maps and properties that are preserved under continuous deformation. It turns out that topology has many surprising applications to fields ranging from economics to combinatorics to data science. In this short course, we will survey some applications of topology to problems of fair division (of cakes, rent, and more!). No prior knowledge of topology will be assumed.

Homework: None

Class format: Interactive lecture

Prerequisites: None.

Congruences and Observables (♳♳, *Steve Schweber*, TW[ΘFS])

A *congruence* on a mathematical structure is, roughly speaking, an equivalence relation that plays nicely with all the stuff we care about. For example, if our starting structure is the integers with addition and multiplication singled out as important (You may have heard the phrase 'the *ring* of integers'.) then the relation '$a \equiv b \mod 12$' is a congruence. Congruences are exactly the equivalence relations we can meaningfully mod out by—e.g. modding out the integers by $\mod 12$ equivalence gives us, well, the integers modulo 12, usually denoted $\mathbb{Z}/12\mathbb{Z}$—and this is why we care about them.

In this class I want to tell a short mathematical story I learned only fairly recently. We usually think about congruences as being built from below: For example, mod 12 equivalence is the smallest equivalence relation satisfying $x \sim x + 12$. However, there is also a way to build congruences from *above*. It turns out that this is exactly what's going on in *combinatorial game theory*, where we assign values to games in a way that comes from—but does more than simply answer—the question of who should win the game (with perfect play).

We'll start out with the definition of *game values*. Then we'll see how this is a special case of a more general abstract procedure that we can apply to *any* algebraic structure, often with surprising results. Finally, we'll end with some open questions—ones I've tried to solve, without success!

Homework: Recommended

Class format: Lecture

Prerequisites: None. In particular, although I mentioned rings and combinatorial game theory in the blurb, you don't need to be familiar with any of this—we'll build it all from scratch!

Cubic Curves (♳♳♳, Mark, [TWΘFS])

A curve in the xy-plane is called a *cubic curve* if it is given by a polynomial equation $f(x, y) = 0$ of degree 3. Compared to conic sections (which have degree 2), at first sight cubic curves are unpleasantly diverse and complicated; Newton distinguished more than 70 different types of them, and later Plücker made a more refined classification into over 200 types. However, as we'll see, by using complex numbers and points at infinity we can bring a fair amount of order into the chaos; in fact, cubic curves have some elegant and excellent properties. One of those properties in particular, which is about intersections, will allow us to prove a beautiful theorem of Pascal about hexagons and conic sections, and it will also let us define a group structure on any cubic curve.

Well, almost. We may have to leave out a singular ('bad') point first, but a cubic curve has at most one such point (which may be well hidden; for example, $y = x^3$ has one!),and most of them don't have any. Cubic curves without singular points are known as *elliptic* curves, and they are important in number theory, for example in the proof of the Fermat–Wiles–Taylor theorem (a.k.a. 'Fermat's Last Theorem]), and in cryptography. However, in this week's class we probably won't look at that aspect at all, and no knowledge of number theory (or even groups) is required. With any luck, along the way you'll pick up some ideas that extend beyond cubic curves, such as how to deal with points at infinity (using "homogeneous coordinates"), what to expect from intersections, and where to look for singular points and for inflection points.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: A bit of differential calculus, probably including partial derivatives; complex numbers; a bit of experience with determinants.

Structure and Randomness (♘♘♘, Ce, TWΘFS)
A huge graph can look impossibly complicated up close. But zoom out far enough, and most of it begins to resemble random noise arranged in a bounded number of blocks. This is the philosophy behind Szemerédi's regularity method: every sufficiently large graph has a low-resolution picture that captures almost all of its behavior. We will learn how to recognize pseudorandom pieces, measure structure using an energy that can increase only so many times, and count small patterns inside an enormous graph. Our main destination is the triangle removal lemma: a graph with very few triangles can be made triangle-free by deleting very few edges. We will then translate this result into arithmetic and prove Roth's theorem that every sufficiently dense set of integers contains a three-term arithmetic progression. The constants will be horrifying; the ideas will be beautiful. In particular, if you attended Bowen's Week 2 class and would like to see a very different route to Roth's theorem, you may find this class interesting.

Homework: Optional

Class format: Interactive Lecture

Prerequisites: Basic graph theory, including bipartite graphs and triangles. You should be comfortable with counting edges within and between sets of vertices.

Hadwiger's conjecture (♘♘, *Pesto*, TWΘFS)
Until 1976, the biggest open problem in graph theory was the *Four-Color Theorem*, that every planar graph's vertices can be divided into four independent sets ("colors", sets containing no edges). Now that that's proven, what's the new biggest open problem in graph theory? The answer is probably a generalization of the Four-Color Theorem called Hadwiger's conjecture.

In a planar graph, the vertices cannot be divided into $k+1 = 5$ *connected* sets with an edge between each pair of the connected sets.

In fact, all graphs with that property (not just planar graphs) can be colored with $k = 4$ colors. Hadwiger's conjecture is that, if you replace $k = 4$ with any positive integer, that statement is still true. So, the $k = 4$ case of Hadwiger's conjecture is already harder than the Four-Color Theorem, but we'll prove it (starting from the Four-Color Theorem) and also talk about an alternate version of Hadwiger's conjecture which is proven.

Homework: Recommended

Class format: Interactive lecture with some problem-solving time

Prerequisites: None

10:10 Classes

Fourier Analysis but Actually Physics (♘♘♘, *Alan Chang* and Laithy, TWΘFS)
What do a vibrating guitar string, a quantum particle trapped in a box, a cooling metal ring, and the circle have in common? Fourier series!

In this sequel to an introductory Fourier series class, we will use the complex expansion

$$f(x) = \sum_{n\in\mathbb{Z}} c_n e^{2\pi n x}$$

as a machine for turning difficult problems into simple ones, one frequency at a time. We will see why strings vibrate in harmonics, solve the Schrödinger equation for a particle in a box, and explain mathematically why heat flow erases sharp edges and smooths everything out.

Fourier analysis makes a surprising jump from physics to geometry. We will prove Wirtinger's inequality and use it to show the isoperimetric inequality—among all closed curves with a fixed perimeter, the circle encloses the most area: $4\pi A \le P^2$.

If time permits, we will study other applications of Fourier series in physics.

Homework: Recommended

Class format: Interactive lectures

Prerequisites: A Fourier series class; comfort with analysis and single variable calculus; comfort with partial derivatives.

Machine Geometry (♧♧♧, Misha, TWΘFS)

Once upon a time (in the mid-90s) people still weren't sure if computers could prove theorems. When it came to Euclidean geometry, mathematicians knew how to turn claims about lines and circles into hideous polynomial equations (but let's not get into that). However, it was an open question whether a computer could prove a theorem in the same way as a human would, without access to human intuition about geometric diagrams.

Then, using a few clever theorems about areas and related quantities, a method was discovered that let computers handle hundreds of theorems by systematically eliminating points, producing short and (from a certain point of view) elegant proofs. This is what we will learn about in this class.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

The First Rare Group of Intermediate Growth (♧♧♧, Neelam, TWΘFS)

In 1968, Milnor asked a very naïve question: Is there a group whose balls of radius n (These are the group elements that can be written with at most n generators.) grow faster than any polynomial in n but slower than any exponential in n? In 1980, Rostislav Grigorchuk constructed such a mathematical monster. The Grigorchuk group is a mathematical paradox: it is finitely generated but not finitely *presented*; it is an infinite group where every element has finite order; and it grows at a rate that is neither polynomial nor exponential, but sits mysteriously right in the middle.

Over these four days, we will get our hands dirty with this counterexample to everything you thought you knew about group theory. We'll skip the dry, abstract definitions and build the group visually by looking at how it shuffles the branches of an infinite binary tree. By tracing simple paths down the tree, we'll see exactly how a few basic, recursive rules can create a fractal-like structure that completely rewrote the rules of geometric group theory.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Introductory group theory (comfort with normal subgroups, homomorphisms, and quotient groups; and familiarity with the free group F_2).

Trail Mix (♧♧ → ♧♧♧, Mark, TWΘFS)

Is your mathematical hike getting a little too strenuous? Would you like to relax a bit with a class that offers an unrelated topic every day—so you can pick and choose which days to attend—and that does not expect you to do homework? If so, some Trail Mix may be just what you need to regain energy! Individual descriptions of the four topics follow.

Day 1: The Catalan Numbers (♧♧, *Prerequisites*: None.)

What's the next number in the sequence $1, 2, 5, 14, \ldots$? If this were an "intelligence test" for middle or high school students, the answer might be 41; that's the number that continues the pattern in which every number is one less than three times the previous number. If the sequence gives the answer to some combinatorial question, though, the answer is more likely to be 42. We'll look at a few questions that do give rise to this sequence (with 42), and we'll see that the sequence is given by an elegant formula, for which we'll see (at least an outline of) a lovely combinatorial proof.

Day 2: Wedderburn's Theorem (♔♔♔, *Prerequisites*: Some group theory and some ring theory; familiarity with complex roots of unity would help.)

Have you seen the quaternions? They form an example of a division ring that isn't a field. (A division ring is a set like a field, but in which multiplication isn't necessarily commutative.) Specifically, the quaternions form a four-dimensional vector space over $\mathbb{R}$, with basis $1, i, j, k$ and multiplication rules

$$i^2 = j^2 = k^2 = -1, \quad ij = k, \quad ji = -k, \quad jk = i, \quad kj = -1, \quad ki = j, \quad ik = -j.$$

Have you seen any examples of finite division rings that aren't fields? No, you haven't, and you never will, because Wedderburn proved that any finite division ring is commutative (and thus a field). In this class we'll see a beautiful proof of this theorem, due to Witt, using cyclotomic polynomials (polynomials whose roots are complex roots of unity).

Day 3: A Tour of Hensel's World (♔♔→ ♔♔♔, *Prerequisites*: Some experience with the idea of a convergent series.)

In one of Euler's less celebrated papers, he started with the formula for the sum of a geometric series:

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1 - x}$$

and substituted 2 for x to arrive at the apparently nonsensical formula

$$1 + 2 + 4 + 8 + \cdots = -1.$$

More than a hundred years later, Hensel described a number system in which this formula is perfectly correct. That system and its relatives (for each of which 2 is replaced by a different prime number p), the p-adic numbers, are important in modern mathematics; we'll take a quick look around this strange "world".

Day 4: Counting, Involutions, and a Theorem of Fermat (♔♔, *Prerequisites:* None.)

Involutions are mathematical objects, usually functions, that are their own inverses. Involutions show up with some regularity in combinatorial proofs; in this class we'll see how to use counting and an involution, but no number theory in the usual sense, to prove a famous theorem of Fermat about primes as sums of squares. (Actually, although Fermat stated the theorem, it's uncertain whether he had a proof.) If you haven't seen why every prime $p \equiv 1 \pmod 4$ is the sum of two squares, or if you would like to see a relatively recent (Heath–Brown 1984, Zagier 1990), highly non-standard proof of this fact, consider taking this class.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Varies by day—see separate descriptions above.

What they Don't Teach You in a Game Theory Course (♔♔, Nikita, TWΘFS)

In our game theory course we developed the standard theory with matrices and equilibria. This time we ask questions a matrix cannot answer. When a game has a hundred equilibria, how do actual humans pick one—for example, where would you meet a stranger in New York City tomorrow if you couldn't communicate? How should you bid at an auction, and how can an auction house run a design under which lying is pointless? Is there a fair way to split a treasure—and can "fair" be a theorem? Why, mathematically speaking, are ants so devoted to their sisters?

Topics will include: first and second price auctions; Schelling points; common knowledge and epistemic game theory; the Nash bargaining problem; cooperative games and the Shapley value; and Hamilton's rule. There will be bids, attempts at reading minds, and coalitions. 'What They Teach You in a Game Theory Course' is useful but not required. You should still be willing to take tiny toy examples far too seriously.

Homework: Optional

Class format: Lecture

Prerequisites: Knowing what Nash Equilibrium is.

1:05 Colloquia

Projections of Random Fractals (*Alan Chang*, Tuesday)
Random fractals are fractals built by repeated random choices. What do their projections look like? This question turns out to have connections to other areas of mathematics, including Kakeya sets (i.e., sets of area zero containing a line segment in every direction) and percolation on trees (i.e., what happens when we randomly remove edges from a tree?).

Why Fractional Calculus isn't a thing (*Steve Schweber*, Wednesday)
We know how to iterate the derivative operation $f \mapsto \frac{d}{dx}f$ a positive integer number of times, and even - allowing for some non-uniqueness around that pesky constant of integration - a negative integer number of times. But what about a *fractional* number of times? What's the one-halfth derivative of e^x, for example?

It turns out that you **can't** do this - there is simply no half-derivative operator!

Now some of you might be thinking "But wait Steve, I just googled 'fractional calculus half derivative' and I got tons of hits. What the heck?" Indeed, there is a notion of "half derivative." In fact, there are *several* notions of "half derivative." The problem is that none of these is really "the" half-derivative, for the simple reason that a couple basic properties about the derivative rule such a thing out completely: there is no half-derivative operator which is both *linear* and *local*.

This is Peetre's theorem! I'll explain what exactly it says, how to prove it, and what - nonetheless - fractional calculus is up to.

The Cayley–Bacharach Theorem (*Jakub Witaszek*, Thursday)
Algebraic geometry is a branch of mathematics that studies geometric shapes described by polynomial equations. Familiar examples include circles and parabolas, as well as more complicated curves such as $y^2 = x^3 + 1$.

In my lecture, I will discuss one of my favourite theorems about curves: the Cayley–Bacharach theorem, which offers striking insights into how two curves intersect.

We will then apply this theorem to classical Euclidean geometry, using it to prove the theorems of Pascal and Pappus.

Time permitting, I will also discuss other theorems in Euclidean geometry that can be proved using methods from algebraic geometry.

Future of You (Staff, Friday)
We shall have informative discussions about what your future looks like. Some topics include: academic or non-academic careers, colleges and admissions, activities you can do in high-school (taking a gap year, skipping a grade, forming a math circle) etc.

What happens at a research conference? (Glenn, Saturday)
For most of you reading this, I'm probably in Europe right now at a research conference. I'm presenting research at a conference in my area, which is about solving the Boolean satisfiability problem: Given a Boolean function like $f(x, y, z) = (x \vee y) \wedge \neg z$, is there any true/false assignment to the variables on which the function is true? I'll introduce to you my research area, share a few highlights from research that I'm seeing myself now, and generally show you what it's like to go to one of these conferences.

2:10 Classes

Continuum Hypothesis (week 2 of 2) (♘♘♘♘♘, Susan, TWΘFS)
This is a continuation of Continuum Hypothesis week 1.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Continuum Hypothesis Week 1.

Differential Forms and Stokes' Theorem (♘♘♘♘♘, Laithy, TWΘFS)
Differential forms are tiny geometric measuring devices: 1-forms measure infinitesimal displacement, 2-forms measure infinitesimal "oriented" area, and k-forms measure infinitesimal "oriented" k-dimensional volume. They are the natural—and, in a precise sense, correct—objects to integrate over curves, surfaces, and higher-dimensional manifolds.

We will define differential forms on smooth manifolds in $\mathbb{R}^n$ and explore their surprisingly rich algebra. In particular, the space $\Omega^k(M)$ of k-forms form "modules" over the ring $C^\infty(M)$; all differential forms together form a "graded commutative algebra" under the wedge product; and the "exterior derivative" turns this into a "differential graded algebra". Don't worry if that sentence currently sounds like wizardry: every term in quotations will be defined, and by the end of the class it will all mean something to you.

Everything will culminate in a proof of the generalized Stokes' theorem,

$$\int_M d\omega = \int_{\partial M} \omega,$$

which is the many-dimensional descendant of the Fundamental Theorem of Calculus that unifies Green's theorem, the divergence theorem, and the classical Stokes' theorem.

Warning: this will be a very challenging, fast-paced, proof-heavy class, with plenty of abstraction and high-dimensional thinking. A potential Week 5 follow-up, De Rham Cohomology, will use these ideas to detect holes in spaces.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Linear algebra (vector spaces, linear independence, basis); and multivariable calculus (partial differentiation, integration).

Error Correcting Codes (♘♘ → ♘♘♘, Narmada and Riley, TWΘFS)
Error-correcting codes are a very useful application of linear algebra and modular arithmetic. When asked by his high-school students in his real life about real-world applications of math, Riley would love to say error-correcting codes, but the math is generally not within reach for his students. Luckily, at Mathcamp it is! We will spend some time on the general theory of error correcting codes and then specialize to Reed–Solomon codes (which some of you may have seen in Eric's class on QR codes). During the last day, you will have time and guidance to program a Reed–Solomon encoder/decoder.

Homework: Recommended

Class format: Interactive lecture, with a day of coding.

Prerequisites: Introductory linear algebra: know what subspaces, bases, and dimension mean; know what the rank of a matrix is; be comfortable multiplying matrices using the vector dot product. Basic modular arithmetic: in particular, know that nonzero residues have multiplicative inverses in $\mathbb{Z}/p\mathbb{Z}$.

Lebesgue vs. Baire (♧♧♧, Maya, TWΘFS)
What makes a subset of the reals "small"? Well, how about if it's finite? That's definitely one way of being small, but there are other ways. Two beautiful and rich notions of smallness come from the respective notions of Baire category and Lebesgue measure, and it turns out (yes, that phrase again) that they mirror each other in some very cool ways. This course is about that mirroring; we will start by going over both Baire category and Lebesgue measure, so it's fine if you aren't familiar with them already. Here's a rough outline of the days:

Day 1: The idea of "small sets", and definitions of being small with respect to Baire category and Lebesgue measure;
Day 2: The notions of being a measurable set and a set with the Baire property;
Day 3: Finding bad sets inside good ones;
Days 4+5: Fubini's theorem and its category analogue, and an indication of why these analogues exist.

Homework: Recommended

Class format: Interactive lecture, with some things for you to do

Prerequisites: You should definitely be comfortable with the following notions in $\mathbb{R}$: open and closed sets, the closure of a set, and completeness. It helps if you are also familiar with the notion of denseness, and know that the rationals are dense in $\mathbb{R}$. If you have taken Riley's course Lebesgue Integration and Ben–Charloutte's course Baire Necessities, some of the material on days 1 and 2 will be familiar to you. +1 chili if you have taken neither and will be seeing these notions for the first time.

Indifference (♧, Purple, TWΘFS)
You might have heard certain people with big microphones saying things like, "We're living in a simulation!" or, "Statistical reasoning suggests that the world is going to end soon!" Arguments of this kind often draw on the work of futurist philosophers like Nick Bostrom and Eliezer Yudkowsky. They almost always rely on the principle of indifference, which (roughly) states that, absent other evidence, we should assign equal credence to all subjective states consistent with our subjective experience.

The goal of this class is to discuss this principle. What exactly is it saying? What are some of its implications? Why might we accept or reject it?

Homework will consist of short readings from the philosophical literature. It will be strongly recommended, though not strictly required to follow the course.

Homework: Recommended

Class format: Discussion

Prerequisites: None.

3:10 Classes

Advanced LaTeX (♧, Allison and Glenn, TWΘFS)
Have you ever Googled a LaTeX formatting question, only to get a magical TeX StackExchange user post a solution full of macros, expressions like `\makeatletter`, and other wizardry that works but you don't understand how? (Or maybe, in this modern age, asked an LLM for help with similar results?)

Have you ever compiled a LaTeX document and received cryptic errors and warnings like "Underfull `\hbox` (badness 10000)"? How do you fix these, and moreover, how does the box layout system of LaTeX actually work?

If you want the answers to these questions and more, come to learn about advanced techniques in LaTeX!

Homework: Optional

Class format: Interactive lecture

Prerequisites: You should know basic LaTeX, such as how to write various equations. The more experience you have struggling with LaTeX, the more interesting this course may be.

PS: MC (♧♧♧♧, *Pesto*, TWΘFS)
Polynomials are a frequent topic of olympiad-style competitions, since there are many and interesting problems using them. For instance, if $a_1, \ldots, a_n$ are distinct real numbers, find a closed-form expression for

$$\sum_{1\le i\le n} \prod_{1\le j\le n, j\ne i} \frac{a_i + a_j}{a_i - a_j}.$$

Don't see the polynomials? Come to class and find them.

Homework: Optional

Class format: A few minutes of lecture about problem-solving techniques; then most of class time is spent working on olympiad-style problems (individually or in groups) and discussing their solutions.

Prerequisites: Linear algebra: be able to write at least two bases for the vector space of polynomials of degree at most 3 in x.

Rings of Continuous Functions (♧♧♧♧, *Steve Schweber*, TWΘFS)
Suppose we have a topological space X. The set of continuous functions from X to [insert fixed target space] is always a natural thing to consider, but gets particularly exciting if the target space is the real numbers $\mathbb{R}$. Since $\mathbb{R}$ is algebraic as well as topological, we now don't just have a set of continuous functions, we have a ring: continuous functions can be added and multiplied (pointwise). This ring is denoted $C(X)$; we also have the related object $C^*(X)$ consisting of all bounded continuous functions $X \to \mathbb{R}$; this would be a ring except for the small problem that it doesn't generally have a multiplicative unit. Still pretty good though.

At first glance it may seem that we lose lots of information in passing from X to $C(X)$ or $C^*(X)$. As it happens, this is not the case. This class will explore just how informative these r(i)ngs are, and—as usual with the nonsense I do—run into some weird set theory problems along the way!

(This is of course not the only way algebra and topology get together—witness algebraic topology, for example, which is related but very distinct from this problem—but it is my favorite!)

Homework: Recommended

Class format: Lecture

Prerequisites: Basic point-set topology (space, subspace, continuous map) and ring theory (subring, quotient ring, ideal). You do not need to be familiar with non-commutative rings or quotient topologies, though.

Stupid Games on Infinite Graphs (♧♧, Della, TWΘFS)
I'm Michael, and I took this class last year. Della didn't write this blurb so now I have to. Here's what I remember:

- I think it was kind of interesting.
- We played some games, except not really. (We moved one step at a time on a graph until we got stuck.)
- The games were fun, except not really. (Infinite games tend to go on for too long.)
- Della lectured about the games and how they encode some math, except not really.
- There was an infinity. And then a bigger infinity. And then an even bigger one. And so on until we all left. (Della kept talking.)
- I don't think there was homework. Maybe I just didn't do it.

So should you come to the class? If you're interested in exploring games with surprising depth, learning about big sets, and haven't seen ordinal numbers before, then sure!

Homework: Optional

Class format: IBL

Prerequisites: None.

Postrequisites: Transfinite induction (if you're comfortable with it, this class is not for you. I'm happy to give you a copy of the problems in TAU.)

The Chevalley–Warning Theorem: Applications and New Directions (♧♧♧, *Jakub Witaszek*, TWΘFS)

I first encountered the Chevalley–Warning theorem as an undergraduate, and it quickly became one of my favourite results in elementary number theory. The theorem gives a striking answer to a fundamental question: what can be said about the number of solutions to a system of polynomial congruences modulo a prime p? Its proof is remarkably elegant, and its consequences extend well beyond number theory. In particular, it has beautiful applications in combinatorics, including the study of regular graphs.

In this course, we will study the statement and proof of the Chevalley–Warning theorem and explore some of its combinatorial applications. We will also discuss solutions modulo p^n and, if time permits, polynomial equations over finite fields. I will conclude by describing some results from my ongoing research work that offer new perspectives on the theorem.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: The course assumes basic familiarity with modular arithmetic and Fermat's little theorem.

Why We Care About the Menger Sponge (♧♧♧, Ben, TWΘFS)

Fractals often come up in pop math for a very simple reason—they look cool. While coolness is a very interesting property, which perhaps justifies interest in the wide family of fractals, we might still wonder why mathematicians come up with them in the first place. *In fact* most fractals come up in mathematical research for other properties.

You may have heard of the *Menger sponge*, a three-dimensional fractal[1] that looks sort of like a weird sponge, if you're very creative in how you look at things. This class will investigate a particular property of the Menger sponge, which—conveniently!—is the first property that brought it to the attention of modern mathematics.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: It will help if you know what a metric space is, but is not strictly necessary.

[1]Er—the "dimension" of a fractal is a weird concept. Let's say "a fractal that is most convenient to imagine in three-dimensional space." The dimension of this one is—well, we'll see.