CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2025

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9:10 Classes

Algorithms in Number Theory (🌶🌶, Misha, TWΘFS)
This is a “hands-on” introduction to number theory. I will introduce you to ideas like the Euclidean algorithm, inverses and exponents in modular arithmetic, the Chinese remainder theorem, the theory of quadratic residues, and maybe more—and along the way, I’ll make sure that the theory is backed up by the ability to do calculations with it.

Many of these calculations are traditionally done by computer: you probably don’t want to try to prove *by hand* that 1,123,465,789 is prime, but 1,123,456,987 is not. Optionally, at TAU from 3pm to 4pm, you can learn how to make a computer do these things for you.

Should you take this class based on how much number theory you know?

- If you don’t feel 100% comfortable with topics like divisibility or modular arithmetic, this class will get you up to speed, but if that’s all you want to take it for, I’d happy to just catch you up on these outside of class, instead: find me at TAU or in the evening.
- If you want to take Nic’s class this week, or Eric or Viv’s classes in week 3, but don’t have the background knowledge for them yet, this class is a good way to get it.
- If you’ve already seen topics like Fermat’s little theorem or the Chinese remainder theorem, you know a bit more than I will assume, but you still won’t be bored.

Homework: Optional

Class format: In the 10am class block, we’ll be doing ordinary interactive lecture as in many other Mathcamp classes.

There is an optional computer component to this, in the computer lab from 3pm to 4pm (during TAU), where I will show you how to teach a computer to do cool tricks using the number theory we learned at 10am. This is not required for you to follow along in class, and if you happen to know enough number theory already, you can probably join us in the computer lab without coming to class.

Prerequisites: Essentially none, but if statements like “$9 \equiv 16 \pmod{7}$” mystify you, you should mentally add a chili to this class, since you’ll be learning to become comfortable with them at the same time as you learn other topics.

Required for: The Number Theory of Quadratic Forms (W1)

Differential Geometry of Surfaces (🌶🌶🌶🌶, Laithy, TWΘFS)
In this class, we’ll develop a rigorous calculus-based geometric theory of 2-dimensional surfaces living in 3-dimensional space, like spheres, tori, cylinders, the Möbius strip, etc. We will explore not just

how they look from the outside, but what geometry feels like from within, as if you were living on the surface.

We'll begin by defining precisely what a surface is in $\mathbb{R}^3$, and then explore both its intrinsic geometry (what you can measure without stepping off the surface, like distances and angles) and its extrinsic geometry (how the surface bends and twists in space). We'll also distinguish between local properties (which depend on the behavior near a point) and global properties (which depend on the surface as a whole). Along the way, we'll encounter: The first and second fundamental forms, parallel transport, geodesics (the straightest possible paths), the Gauss Curvature, and Gauss' Theorema Egregium (the remarkable theorem that ties it all together).

This class is a blend of calculus, visualization, and deep geometric ideas—and a chance to explore what it means to do math on curved spaces.

Homework: Recommended

Class format: Interactive Lectures

Prerequisites: Calculus (differentiation and integration). Basic elements of linear algebra (matrix multiplication, linear independence). Multivariable calculus is recommended (partial differentiation, derivatives of maps from $\mathbb{R}^2$ to $\mathbb{R}^3$).

Multivariable Calculus Crash Course (🌶🌶🌶, Mark, TWΘFS)
In real life, interesting quantities usually depend on several variables (such as the coordinates of a point, the time, the temperature, the number of campers in the room, the real and imaginary parts of a complex number, ...). Because of this, "ordinary" (single-variable) calculus often isn't enough to solve practical problems. In this class, we'll quickly go through the basics of calculus for functions of several variables. As time permits, we'll look at some nice applications, such as: If you're in the desert and you want to cool off as quickly as possible, how do you decide what direction to go in? What is the total area under a bell curve? What force fields are consistent with conservation of energy? One reason, and maybe the best reason, to take this crash course right now rather than waiting until you encounter the material naturally after BC calculus and/or in college, is to be able to take the course on functions of a complex variable (which have many amazing features) that starts in week 2.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Single-variable calculus (both differentiation and integration); a basic understanding of vectors would also help.

Required for: Functions of a Complex Variable (W2); The shape and soul of a surface: the Gauss Bonnet theorem (W2); Magic of Harmonic Functions (W3); Functions of a Complex Variable (W3); Einstein's theory of gravity 1: Special relativity (W3); Einstein's theory of gravity 2: General relativity (W4)

On Beyond i (🌶, Steve, TWΘFS)
I like numbers! There are lots of real numbers, but what if I want even more numbers? This class is a two-part introduction to a method for building really big number systems.

The first part of this class will be an introduction/refresher to complex numbers. Here we'll be focusing, not on quadratic equations, but on **cubics**, which historically were actually how complex numbers were introduced.

The second part of this class will focus on a more silly question: why not add **more** square roots of -1? Or, extra square roots of positive 1? Or something else? It turns out that there is a general trick for making almost any number system bigger, giving rise to the **quaternions**, **octonions**, and more. Of course, this also does many horrible things - for example, multiplication of octonions is not even associative! - and we'll see these too.

Homework: Optional
Class format: Lecture
Prerequisites: None

The Other Other Analytic Number Theory (Modular Forms) (🌶🌶🌶🌶, *Dave Savitt*, TWΘFS)

It is sometimes said that there are five elementary operations in arithmetic: addition, subtraction, multiplication, division, and modular forms.

This two-week class will be a hands-on introduction to modular forms. We'll start the first week with some vignettes about infinite products from the work of Euler: his formula for the sine function

$$\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2}\right)$$

and the pentagonal number theorem

$$\prod_{n=1}^{\infty} (1 - q^n) = \sum_{k \in \mathbb{Z}} (-1)^k q^{k(3k-1)/2}.$$

(Think for a moment about how amazing this formula is, how much unexpected cancellation has to happen for the right-hand side to have terms only in degrees $k(3k-1)/2$.). We'll also introduce the Bernoulli numbers and some of their basic properties.

Then we'll fast forward to the 19th century to see how these are all tied together by the theory of modular forms, with some amazing applications to arithmetic. For example, defining

$$\Delta = q \prod_{n=1}^{\infty} (1 - q^n)^{24} = \sum_{n=1}^{\infty} \tau(n) q^n$$

we'll prove the congruence $\tau(n) \equiv \sigma_{11}(n) \pmod{691}$, where $\sigma_{11}(n)$ denotes the sum of the 11th powers of the divisors of n. If time permits we'll prove Jacobi's four-square theorem, that the number of ways of writing a positive integer n as a sum of four squares is 8 times the sum of the divisors of n that are not divisible by 4. (If time doesn't permit, the proof will be in the lecture notes.)

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Some comfort with the language of analysis (limits, convergence, absolute convergence) is a must. Probably this is all that's needed for the vignettes at the start of the class — I'll try to minimize the prerequisites that are needed at first. But after that, you should be happy with the sentence "Let U be an open subset of $\mathbb{C}$," the expression $e^{2\pi i z}$ should be meaningful to you, and you should either be familiar with, or willing to accept, some basics of complex analysis (which we will review).

10:10 Classes

Counterexamples to the Fundamental Theorem of Calculus (🌶🌶🌶🌶, Ben Dees, TWΘFS)
OK, so the course title is clickbait—the whole point of theorems is that they don't have counterexamples. But what *exactly* does the Fundamental Theorem of Calculus say? I'll leave that as a floating question for now. What, then, is this course about?

In this course, we will construct a function $f\colon [0,1] \to$ so that f is differentiable *everywhere*, but its derivative f' is *not Riemann-integrable*. That is, we shouldn't write something like

$$\int_0^1 f'(x)dx = f(1) - f(0)$$

for this function if we want the squiggly thing on the left there to mean a Riemann integral.

To figure out why this is the case, we'll need to dig deep into what Riemann integrals are, which will require us to build up some of the analysis background needed to talk about integration formally. So, the first few days will be a bit of a whirlwind introduction to analysis, closing out with a precise definition of Riemann integrability. After that, we'll prove a beautiful theorem of Lebesgue, which gives a convenient way to check if a function is Riemann integrable, and we'll use *that* theorem as a helpful recipe for cooking up our no-good, very bad, not-Riemann-integrable derivative.

Time permitting, we will delve further into the dark spaces where terrifying functions lurk, and find derivatives *that are even worse.*

Homework: Recommended

Class format: Interactive lecture

Prerequisites: The only prerequisite is some familiarity with differentiation and integration. In particular, analysis is *not* a prerequisite. If you have seen some analysis or point-set/metric topology, you may be a bit "ahead" for the first few days.

Elements of a Classical Chess Engine (🌶🌶, Riley, TWΘFS)
Calling all chess players! If you have ever been mesmerized with the way that Stockfish brutally demolishes any human foolish enough to play it, this may be the class for you. If instead you are just a speed demon programmer and love seeing a fast algorithm, you may be interested as well. Or, if you just want to see some magic, there may be some of that, too... We will discuss several components of how computers play chess, focusing on the clever engineering decisions and optimizations that let a computer look at tens of millions of chess positions per second.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Having some familiarity with C-style languages is encouraged but definitely not required! It may also help to have some chess experience.

Finite Field Trip (🌶🌶, Eric, TWΘFS)
Field trips are starting early this year! Join me on an adventure through the world of finite fields. In field trip terms this will be more "curated tour" than "immersive experience": after orienting ourselves as to what finite fields even are, we'll see some standard and not-so-standard ways of constructing finite fields, we'll touch a little bit on the Galois theory of finite fields and the all important Frobenius map, and we'll end with a couple of applications to both the real world (QR codes) and the world of pure math (the Weil conjectures). Our class time will sacrifice proving some of the "basic" facts for the sake of covering more of the landscape, though the homeworks will have maps for all the side trails we skipped in class.

Homework: Recommended

Class format: Mostly IBL style worksheets (your "trail maps") interspersed with mini-lectures.

Prerequisites: Two elements of basic number theory are helpful: modular arithmetic at the level of modular units, and the Euclidean algorithm for computing gcds. We'll need a tiny bit of group theory (orders of group elements and Lagrange's theorem), but you can totally pick this up on the way if you're also taking the week 1 group theory class. Consider this class as +1 🌶if you're also learning these prereqs for the first time this week.

Introduction to Linear Algebra (🌶🌶, Narmada, TWΘFS)
Linear algebra is ostensibly the study of linear equations, but that's like calling writing the study of

the alphabet. It's a fundamental area of mathematics that takes our intuition about how geometry works and turns it into a beautiful theory using both algebra and analysis.

This class will give you an introduction to some classic results about finite-dimensional vector spaces, setting you up to take more advanced classes later in camp. Specifically, we'll focus on vector spaces in $\mathbb{R}^n$ and $\mathbb{C}^n$, and linear maps between them. On the very last day, we will learn a little bit about eigenvalues and the wide world of abstract vector spaces that awaits you.

Homework: Recommended

Class format: Lecture

Prerequisites: none

Required for: Hilbert Spaces (over $\mathbb{C}$) - What does $1\frac{1}{2}$ linear mean and why is it so helpful? (W2); Singular Value Decomposition (W2); Arithmetical Structures on Graphs (W2); The shape and soul of a surface: the Gauss Bonnet theorem (W2); QR Factorization (W3); Representation Theory of Finite Groups (W3); Einstein's theory of gravity 1: Special relativity (W3); Einstein's theory of gravity 2: General relativity (W4); Representation Theory of Finite Groups (W4); Continuous Functional Calculus on Hilbert Spaces (over $\mathbb{C}$): We can take the square root of a function now?! (W4); Seasonal Infectious Disease Models (W4)

The Number Theory of Quadratic Forms (, *Nic Ford*, TWΘFS)
Which integers can be written as a sum of two squares? If you've never thought about this question before, it's worth spending a few minutes with it and trying to find some patterns! To help you out, the sums of squares between 0 and 40 are:

$$0, 1, 2, 4, 5, 8, 9, 10, 13, 16, 17, 18, 20, 25, 26, 29, 32, 34, 36, 37, 40.$$

This question will be our starting point, and between the lectures and the exercises we'll hopefully have a complete answer to it by the end of the second day.

From there, we'll go on to study a generalization of it: given three integers a, b, c, which integers can be written in the form $ax^2 + bxy + cy^2$? This expression is called a *binary quadratic form*, and investigating it will take us through the rest of the week.

We won't be able to answer this question in full generality, but our partial efforts to do so will show us how it relates to the problem of finding prime factorization in exotic number systems where factorizations might not be unique and numbers might not have gcd's. This question gives one of the best introductions I know of to a fascinating corner of math called *algebraic number theory*, and after studying it for a bit you should be well set up to dive deeper into it!

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Some number theory will be assumed. If you know how to tell whether an integer a is invertible modulo some other integer n, and you've seen the fact that for any integers m, n it's always possible to write $\gcd(m, n) = am + bn$ for some a, b, then you probably know enough. Feel free to come find me and ask if you're not sure!

At one point we'll also need a tiny bit of linear algebra; if you know how to find the determinant of a 2×2 matrix and have seen how to use it to find the inverse of the matrix, that should be more than enough.

11:10 Classes

Conjugate Gradient (, Kaia, TWΘFS)
You may have had to solve systems of linear equations $Ax = b$ by hand– perhaps by computing the inverse of the matrix A. Perhaps you found these computations rather tedious or annoying. As it

happens, computers don't love doing things this way either! For one thing, it's far too slow to be practical for very large problems.

Can we develop an algorithm that can solve $Ax = b$ in a better way? What do we even mean by better?

This course will examine one such "better" algorithm that's used in practice–Conjugate Gradient, an iterative method that's often applied to large matrices with lots of zeros. Understanding how and why CG works will require us to conceptualize the solution to $Ax = b$ as the minimum of a quadratic form, to think about different notions of orthogonality, and to ask what the eigenvalues and eigenvectors of A tell us about the geometry of our problem and how well we can find a solution. On the last day, we'll look at an application to image processing, and talk about practical considerations like stopping conditions.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Derivatives, dot products, orthogonality, subspaces, bases, eigenvalues and eigenvectors. It would be ideal to have seen diagonalization and changes of basis– there will be some review and a handout at the end of day 1 discussing the linear algebra we'll use on future days.

Extra-Stretchy Rubber Sheet Geometry (🌶, Purple, TWΘFS)

A somewhat famous expression goes: "to a topologist, a coffee cup and the surface of a donut look the same." This is because topologist thinks of geometric objects as living on perfectly stretchy rubber sheets, and you could stretch a rubber donut into a coffee cup (the donut becomes the handle). But like, how is this math?? And in this strange new world, how can we prove that any two spaces are *not* the same?

The primary goal of this course is to introduce the fundamental group, an excellent tool for answering the latter question. (We will not assume any knowledge whatsoever about groups.) Our approach will be heavily pictoral; we will lean on geometric intuition rather than giving formal definitions and proofs. We will aim to develop the proof that a 2-dimensional plane looks genuinely different from 3-dimensional space, even according to topologists. Then we will see how far we can go with this set of ideas!

Homework: Recommended

Class format: interactive lecture

Prerequisites: None.

Generating Functions, Catalan Numbers and Partitions (🌶🌶, Mark, TWΘFS)

Generating functions provide a powerful technique, used by Euler and many later mathematicians, to analyze sequences of numbers; often, they also provide the pleasure of working with infinite series without having to worry about convergence.

The sequence of Catalan numbers, which starts off 1, 2, 5, 14, 42, ..., comes up in the solution of many counting problems, involving, among other things, voting, lattice paths, and polygon dissection. We'll use a generating function to come up with an explicit formula for the Catalan numbers.

A partition of a positive integer n is a way to write n as a sum of one or more positive integers, say in nonincreasing order; for example, the seven partitions of 5 are

5, 4 + 1, 3 + 2, 3 + 1 + 1, 2 + 2 + 1, 2 + 1 + 1 + 1, and 1 + 1 + 1 + 1 + 1.

The number of such partitions is given by the partition function $p(n)$; for example, $p(5) = 7$. Although an "explicit" formula for $p(n)$ is known and we may even look at it (in horror?), it's quite complicated. In our class, time permitting, we'll combine generating functions and a famous combinatorial argument due to Franklin to find a beautiful recurrence relation for the (rapidly growing) partition function. This formula was used by MacMahon to make a table of values for $p(n)$ through $p(200) = 3972999029388$, back when "computer" still meant "human being who does computations".

Homework: Recommended

Class format: Interactive lecture

Prerequisites: Summation notation; geometric series. Some experience with more general power series may help, but is not really needed. A bit of calculus may come in handy, but you should be able to get by without.

Introduction to Group Theory (🌶🌶🌶, *Mira Bernstein*, TWΘFS)

Abstract algebra studies how mathematical objects interact and combine to form new objects. For example, numbers combine via addition or multiplication (among other things); functions combine via composition; individual moves on a Rubik's cube combine into more complicated patterns; knots combine by intertwining. Abstract algebra is what happens when you don't care about the objects themselves, but only about the structure of their interaction.

Groups theory lies at the heart of abstract algebra. It examines a type of interaction that occurs over and over in many mathematical contexts: a *binary, associative operation with an identity and inverses.* (Don't worry if you don't know what that means – we'll explain.) The general results you prove in group theory can be applied to geometry, number theory, combinatorics, topology, physics—basically everywhere! That's why group theory is a prerequisite for so many classes at Mathcamp.

In this introductory class, we will cover the basic definitions of group theory, Lagrange's Theorem, homomorphisms, quotient groups, the First Isomorphism Theorem, and a little bit on symmetries and permutation groups. If you've seen most of these topics before, no need to take this class. If you haven't, come join us for a first foray into the beautiful realm of abstract algebra!

Homework: Required

Class format: Mostly interactive lecture. Because group theory is such a foundational subject and so different from other math you may have seen before, the homework for this class is required: you absolutely can't learn it without doing it.

Prerequisites: None

Required for: Geometric Group Theory (W2); Breaking the axiom of choice (W2); Model Theory (W3); Geometric Group Theory (W3); Representation Theory of Finite Groups (W3); Dirichlet's class number formula (W3); Representation Theory of Finite Groups (W4)

Percolation (🌶🌶🌶🌶, Nikita, TWΘFS)

Imagine a giant random QR code. Can you cross it from left to right by stepping only on the black squares? This simple question lies at the heart of percolation theory, a rich area of probability that explores how global structures emerge from local randomness.

In this class, we'll introduce the basics of percolation, uncover surprising symmetries in random networks, and learn about positive correlations through the beautiful Harris inequality. We'll end by discussing the recently disproven Bunkbed Conjecture, a charming problem that sat open for decades.

Homework: Recommended

Class format: Lecture + problem solving

Prerequisites: Probability, inclusion-exclusion principle

1:10 Classes

3D (🌶, *Po-Shen Loh*, TWΘ[FS])
The standard school curriculum, and math Olympiads, have plenty of coverage of plain (plane) geometry. We'll talk about some things that involve going into another dimension.

Homework: Recommended

Class format: Interactive lecture

Prerequisites: None.

Introduction to Point-set Topology (🌶🌶🌶, Audrey, [TWΘFS])
You may have heard of continuous functions on $\mathbb{R}$, but $\mathbb{R}$ has a good notion of distance this relies on. How can we generalize continuity to allow for situations without any way of having a "nice" distance? We will figure out how to generalize continuity and also explore one of my personal favorite ways that continuous functions preserve structure via Urysohn's Lemma.

Homework: Recommended

Class format: Lecture

Prerequisites: None

Required for: Introduction to Descriptive Set Theory (W2)

Mathcamp Crash Course - 80 min (🌶, Glenn, [TWΘFS])
First time at Mathcamp? This is the class for you! Mathcamp Crash Course is your personal guide to get the most out of all your classes here, and is recommended for all new campers, especially if your mathematical background leans more towards calculations and/or answer-based contests like AMC and AIME, (in contrast to proof-based arguments and Olympiads). We'll practice the fundamentals of mathematical reasoning and logic that other Mathcamp courses throughout the summer will often take for granted.

Here's the key point: At Mathcamp, most math is not really about getting the right *answer*, but rather the more difficult problems of having a correct *explanation* and effectively *communicating* that reasoning to others. In other words, almost all your classes here will be about noticing things that are true, pausing to ask, "Huh, why is that?" and then coming up with a convincing reason. After taking Crash Course, you'll have the right toolbox to start thinking about all those interesting questions.

Concretely, this class will cover: syntax of mathematics, logic, sets, proofs with different logical structures, induction, numbers, functions, relations, cardinality, and infinity. And remember—we're focusing on the why, not the what, so even if you know what all these things are, you might still want to consider taking this class!

(And along the way, since after all, I am a computer scientist, I probably won't be able to help but share a couple of golden nuggets: how programming is secretly just induction, the existence of functions that no computer can solve, and maybe a few more. Absolutely no computer science background expected though, and this definitely isn't the core content!)

Homework: Required

Class format: Group work, with flipped classroom (except first day). That means that there are no homework problems, but you will be expected to read a few pages of notes before coming to class. In class, we'll quickly review what you read, and then jump into solving problems together!

Prerequisites: None.

Stupid Games on Infinite Graphs - 80 min (🌶🌶, Della, TWΘFS)
Draw some dots with arrows connecting them. Pick a dot to start on. Now play a game: on each turn, move across an arrow to another dot. The game ends, and you lose, if there are no arrows from your dot.

We will investigate questions about this game: from some position, is it possible to survive forever? If not, how many turns can you make before getting stuck? To understand the answers, we will be forced to invent the ordinal numbers, and we'll see what this all tells us about Chess on an infinite board.

Homework: Optional

Class format: IBL

Prerequisites: None. If you've seen ordinal numbers before, I'll ask you to set aside what you know so we can start from scratch.

The Not So Ordinary Theory of Ordinary Differential Equations (🌶🌶🌶, Sam, TWΘFS)
Differential Equations (DEs) offer an incredibly powerful and cool framework to study not only the world around us (from Biology to General relativity), but even to answer complex problems in Geometry (such as minimal surfaces) and other areas of pure mathematics. But what happens if I'm studying a DE that models physical phenomena like fluid flow, and I can find more than one solution to my equation? Surely only one of these solutions models what will happen in real life. This raises the question, if I have a solution to a DE, how do I know it's unique? (i.e. that it is the only solution) This is an incredibly difficult question to answer, DEs with different structures require different approaches, in fact for some DEs we don't have a clue. Even for the simplest of cases, that is, Ordinary DEs (ODEs) (DEs where there is only one independent variable), the path to showing uniqueness of solutions requires us to build up a lot of cool analysis tools, which are interesting in their own right, in order to study the problem. From Banach spaces to fixed point theorems, I will take you on a journey that culminates in the proof of the Picard–Lindelöf theorem, the crown jewel of the theory of existence and uniqueness of ODEs. If you love analysis or you enjoy applied maths and have an urge to understand some of the theory behind it, then this is the course for you.

Homework: Recommended

Class format: Interactive lecture and worksheets

Prerequisites: • Definition of open intervals; • A basic understanding of functions; • Definition of sequences and limits of sequences in $\mathbb{R}$; • Definition of continuous functions $f : (a, b) \to \mathbb{R}$; • Definition of the derivative of a function $f : (a, b) \to \mathbb{R}$; • Knowledge of the triangle inequality for $|\cdot|$ on $\mathbb{R}$; • Definition of a vector space.

The Real Numbers (🌶, Maya, TWΘFS)
You've worked with and computed with the real numbers forever, but how are they defined, exactly? You might say the real numbers are all the possible ways you could write numbers with a decimal point. Sure, but then why is 0.9999[...] "the same" as 1? Are there any other pairs that are "the same"?

More importantly, why are we defining the real numbers? We define the natural numbers to count, we define the integers so that addition and subtraction work, we define the rational numbers so that multiplication and division work. The goal of this class is to talk about why we need to go one step further to define the reals.

This class is a first step into real analysis. We will construct the real numbers rigorously and see how the construction interplays with ordering and the arithmetic operations that we take for granted. If you are dissatisfied with your understanding of the real numbers, and want the same clarity about what they are that you have for the rationals, this class is for you!

Homework: Recommended
Class format: Interactive lecture
Prerequisites: None.

Colloquia

The Origins of Set Theory: Cantor and Fourier Series (*Ben Dees*, Tuesday)
One occasionally hears that Georg Cantor first started thinking about set theory while considering a question about Fourier series. While this is true, it can come as a surprise to students of set theory and Fourier analysis alike, since the subjects don't have an obvious synergy.

The question Cantor was trying to answer is this: Suppose that $f(x)$ is some function which can be written as a trigonometric series,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx).$$

Can we write f as a trigonometric series in *two different ways*?

We'll see how Cantor solved this, and why his methods have (to paraphrase the man himself) a necessary and infinite analogue.

The Banach-Tarski Paradox (Narmada, Thursday)
"If you give a mathematician one unit ball, she will partition it into finitely many pieces and reassemble them to form two disjoint unit balls. She will also never stop talking about the proof for the rest of her life." –Stefan Banach or someone

Come see the proof of the Banach-Tarski paradox to learn what the hype is all about! I'll talk about the history of the problem, Hausdorff's original proof that was scooped by Banach and Tarski, and what free groups have to do with all this. If time permits, I will provide a physical demonstration of the paradox.

An Original Way to Approach an Important Constant (*Po-Shen Loh*, Friday)

Prof. Loh will talk about a thing that is unusually missing in this paragraph. His hobby is to think of non-ordinary math proofs, illuminating math insights, particularly visually. This talk will show such an approach for an important constant, which surprisingly is not in any book.

(Much but not all of this talk was in his non-colloquium class at Mathcamp 2024.)