How to Apply

We invite students aged 13 through 18 who love mathematics to apply, regardless of racial, ethnic, religious, or economic background.

We offer two application deadlines. The options are simply for your convenience; we have no preference!

Early Action	Apply to Mathcamp by March 15 →	Decision sent to you on April 1
Regular Action	Apply to Mathcamp by April 15 →	Decision sent to you on May 1

An application to Mathcamp consists of the following:

1) Some basic information about **you and your math background**. We will ask you to describe the math courses you've taken, along with scores and awards from any math competitions you've done.

2) A **personal statement** about your interest in math and what motivates you to apply to Mathcamp.

3) Your solutions to the 2015 **Qualifying Quiz** (see below).

4) **Two recommendation letters**, academic and personal. One letter should speak to your mathematical maturity and readiness for Mathcamp, and the second should address your independence and personal qualities. We are looking for students who will thrive in the atmosphere of freedom and responsibility that characterizes Mathcamp, and who will make a positive contribution to the camp community.

5) The **Request for Financial Aid**, as needed.

To get started, go to: http://www.mathcamp.org/apply

Once you have begun the online application, you may submit your Quiz and recommendations online, by postal mail, or by fax. You may also call to request a paper application. There is no application fee.

Questions? Contact Us! By email: info15@mathcamp.org
Phone/Fax: 888-371-4159 • On the Web: www.mathcamp.org

Cost and Scholarships

We are deeply committed to enabling every qualified student to attend Mathcamp, regardless of financial circumstances.

The full camp fee is $4000 US.

This includes tuition, room, board, & extracurriculars.

20% of our students receive full scholarships!
Mathcamp is FREE for US & Canadian families with household income below $60,000 US.

We also have travel subsidies for those who need assistance covering the cost of transportation to and from camp.

Need-based aid is available for *all* families.

Families with incomes above $60,000 who need assistance will be asked to contribute no more than 3% of their income. There is no set formula: we consider each request on an individual, case-by-case basis. We are committed to meeting 100 percent of demonstrated need for both new and returning students.

Admission to Mathcamp is need-blind for US and Canadian students, so your financial need has no impact on your chances of admission.

International students are also eligible for financial aid, including full scholarships and travel grants.

We *do* take financial circumstances into account for international students during the admissions process because the cost of travel to camp can be very high, and we want to be completely confident that when we admit a student, we are able to meet her or his full financial need. Nonetheless, we admit many international students each year on partial and full scholarships. We also provide travel grants to help new and returning students from all over the world fly to Mathcamp. Please do not let financial need prevent you from applying!

Mathcamp 2015 Qualifying Quiz

Instructions

We call it a Quiz, but it's really a challenge: a chance for you to show us how you approach new problems and new concepts in mathematics. What matters to us are not just your final results, but your reasoning. Correct answers on their own will count for very little: you need to justify all your assertions and ***prove*** to us that your solution is correct. (See www.mathcamp.org/proofs.) Sometimes it may take a while to find the right way of approaching a problem. Be patient: there is no time limit on this quiz.

The problems are roughly in increasing order of difficulty, though the later problems often have some easier parts. We don't expect every applicant to solve every problem. However, the more problems you attempt, the better your chances of being admitted, so we strongly recommend that you try all of them! If you are unable to solve a problem completely, send us the results of your efforts: partial solutions, conjectures, methods – everything counts. None of the problems require a computer; you are welcome to use one if you'd like, but first see a word of warning at www.mathcamp.org/computers.

If you need clarification on a problem, email your question to quiz15@mathcamp.org. You can use books or the Web to look up definitions, formulas, or standard techniques, but any information obtained in this way must be clearly referenced in your solution. Please do not try to look for the problems themselves: we want to see how well you can do math, not how well you can use Google! **Any deviation from these rules will be considered plagiarism and may permanently disqualify you.**

Have fun and good luck!

Problems

(1) An improper fraction p/q (with $p > q$) is written on a chalkboard. A Mathcamper walks by, sees it, and decides to convert it to a mixed fraction, $n\,{}^{r}/_{q}$. She then erases the original.

The next day, another Mathcamper walks by, sees $n\,{}^{r}/_{q}$ written on the board; he decides that it's a multiplication problem, evaluates the product, writes it on the board as ${}^{nr}/_{q}$, and erases the original. Note that if $nr \geq q$ then we once again have an improper fraction.

This misunderstanding keeps repeating, until the result is either an integer or a proper fraction. (After that, everyone walking by leaves it alone.) For example, the sequence might be:

$$^{85}/_{6} \to 14\,^{1}/_{6} \to {}^{14}/_{6} \to 2\,^{2}/_{6} \to {}^{4}/_{6} \to \text{END}.$$

(a) In the example above, we ended up with a fraction that is not in lowest terms. Now suppose each person reduces his or her fraction to lowest terms before writing it down. Can this affect the final result (if not in our example, then perhaps in a different one)?

(b) Suppose the original improper fraction was $n/2$, with n odd. Find, with proof, all values of n for which the final result will be a proper fraction (not an integer).

Note: whatever condition you come up with, you obviously need to show that, if you start with $n/2$ satisfying this condition, you will end up with a proper fraction. But you also need to show that you are not missing anything. In other words, you will need to prove that, for any fraction $n/2$ *not* satisfying your condition, you will end up with an integer.

(c) Prove that, for any denominator $q \geq 2$, there are infinitely many rational numbers p/q for which the process ends with a proper fraction.

(d) (EXTRA) If you feel like playing with this set-up some more and seeing what other results you can derive, please send us anything interesting that you come up with!

(2) Given a triangle ABC, we can "fold it in half" to get a new triangle: pick a vertex, e.g. A, and fold so that segments AB and AC line up. The same can be done from vertex B and vertex C, so there are three different ways to fold a triangle in half (Figure 1).

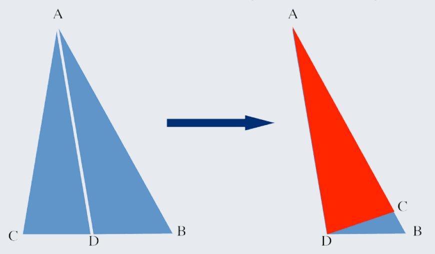


Figure 1: AD is the angle bisector at ∠BAC. If we fold △ABC at vertex A, we obtain △ABD.

(a) If the angles at vertices A, B and C are α, β, and γ, respectively, with $\alpha \leq \beta \leq \gamma$, what are the angles of the three possible triangles that can be obtained by folding ABC in half?

(b) You can fold a 45-45-90 triangle at the right angle to obtain another 45-45-90 triangle. Describe all triangles that can be folded in half to get a similar triangle.

(c) You can fold a 30-60-90 triangle to obtain a 30-30-120 triangle, then fold it again to get another 30-60-90 triangle. Describe all triangles that can be folded in half twice to get a similar triangle.

(d) Prove that no matter how many times you fold a 40-60-80 triangle, you can never get another 40-60-80 triangle.

(3) For a positive integer n, let P_n be the product of all the positive integer divisors of n (including n itself). For example, if $n = 10$, then $P_n = 100$, because $1 \cdot 2 \cdot 5 \cdot 10 = 100$. Suppose I'm thinking of an integer n, but I only tell you P_n. Does this always give you enough information to figure out what n is? If so, prove it. If not, find a counterexample: a pair of integers m and k such that $P_m = P_k$. You might find the following background reading useful: www.cut-the-knot.org/blue/NumberOfFactors.shtml.

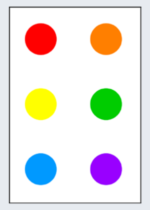
(4) In King Alfonso's royal court, all nobles are either friends or enemies, following the principle "The enemy of my enemy is my friend". (That is, two nobles that share an enemy must be friends, though they can still be friends even if they don't share an enemy.)

For the upcoming royal tennis tournament, the king wants to split up the nobles into pairs of friends. Obviously, if there is an odd number of nobles, pairing them up is impossible. Also, if one noble has no friends at all, then no friendly pairing can be found. But are these the only problems that stand in King Alfonso's way?

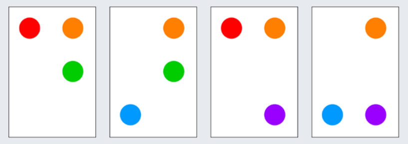
(a) Give an example of a friend/enemy network with an even number of nobles, in which each noble has at least one friend, yet the king still can't pair up the nobles in a friendly pairing.

(b) Describe all friend/enemy networks for an even number of nobles in which the king cannot pair up the nobles into friendly pairs. (Whatever conditions you come up with, don't forget to prove that for all friend/enemy networks that don't satisfy your conditions, a friendly pairing *does* exist.)

(5) The game of ProSet is played with a deck of 63 cards. One of the cards has six dots of six different colors; each of the other cards has some of the dots but is missing others. All the cards are different and each card has at least one dot.



A *proset* is a set of cards that together contain an even number of dots of each color. For instance, the four cards shown below form a proset.



(a) Prove that the entire ProSet deck is itself a proset (i.e. the total number of dots of each color is even).

To play the game, players lay seven cards on the table and compete to see who will be the first to find a proset. (*Just for fun: if you'd like to play the game online, visit* www.mathcamp.org/2015/proset/. *You can also buy a hard copy of the game at* www.zerosumz.com.)

(b) Prove that any set of seven cards is guaranteed to contain a proset.

(c) For $n < 7$, what is the probability that a random set of n cards contains a proset?

(d) Now suppose you play a different version of the game, in which you are only looking for prosets that have exactly three cards. Find, with proof, the smallest n such that any set of n cards is guaranteed to contain a three-card proset.

(7) King Alfonso devises a game to test the intelligence of his royal advisor, Angela. Alfonso tells Angela to gather n of her friends, each of whom will receive a black or white hat. Each friend will be able to see the color of everyone else's hat, but not his or her own. The friends will then be asked to stand in a circle and guess their own hat colors by whispering them to the King. (The King decides the order in which they stand, and they do not hear each other's guesses.) If k out of the n players guess correctly, Alfonso will give Angela and her friends k bags of gold (to divide among themselves as they choose).

Once the rules of the game are announced, the friends are not allowed to talk to each other, so they cannot devise a strategy. Angela (who is not one of the players) must devise a strategy for them: she must write them a letter that will be read aloud by the King. *The letter may not give different instructions to different people*; it must tell everyone the same thing, such as "Guess the color you see more of; if there's a tie, guess black" or "If the hat to you left is the same as hat to your right, guess black; otherwise, guess white." You may assume that Angela's friends always follow her instructions correctly. Note that after reading the letter, King Alfonso knows Angela's instructions to the players and can choose a hat color combination to exploit any weak points in her strategy.

(a) Prove that if $n = 2$, Alfonso is not forced to give out any gold, regardless of Angela's strategy.

(b) Prove that for $n > 2$, Angela can always force Alfonso to give out at least one bag of gold. In other words, Angela can devise a strategy in which at least one player is guaranteed to guess correctly, no matter what assignment of hats Alfonso chooses.

(c) Find and prove an upper bound on the number of bags of gold that Alfonso can be forced to give out. For example, you might suspect that $n - 1$ is an upper bound; in other words, no matter what strategy Angela adopts, Alfonso can always find an arrangement of hats for which at least one player will guess incorrectly. Either prove this bound or find a smaller one.

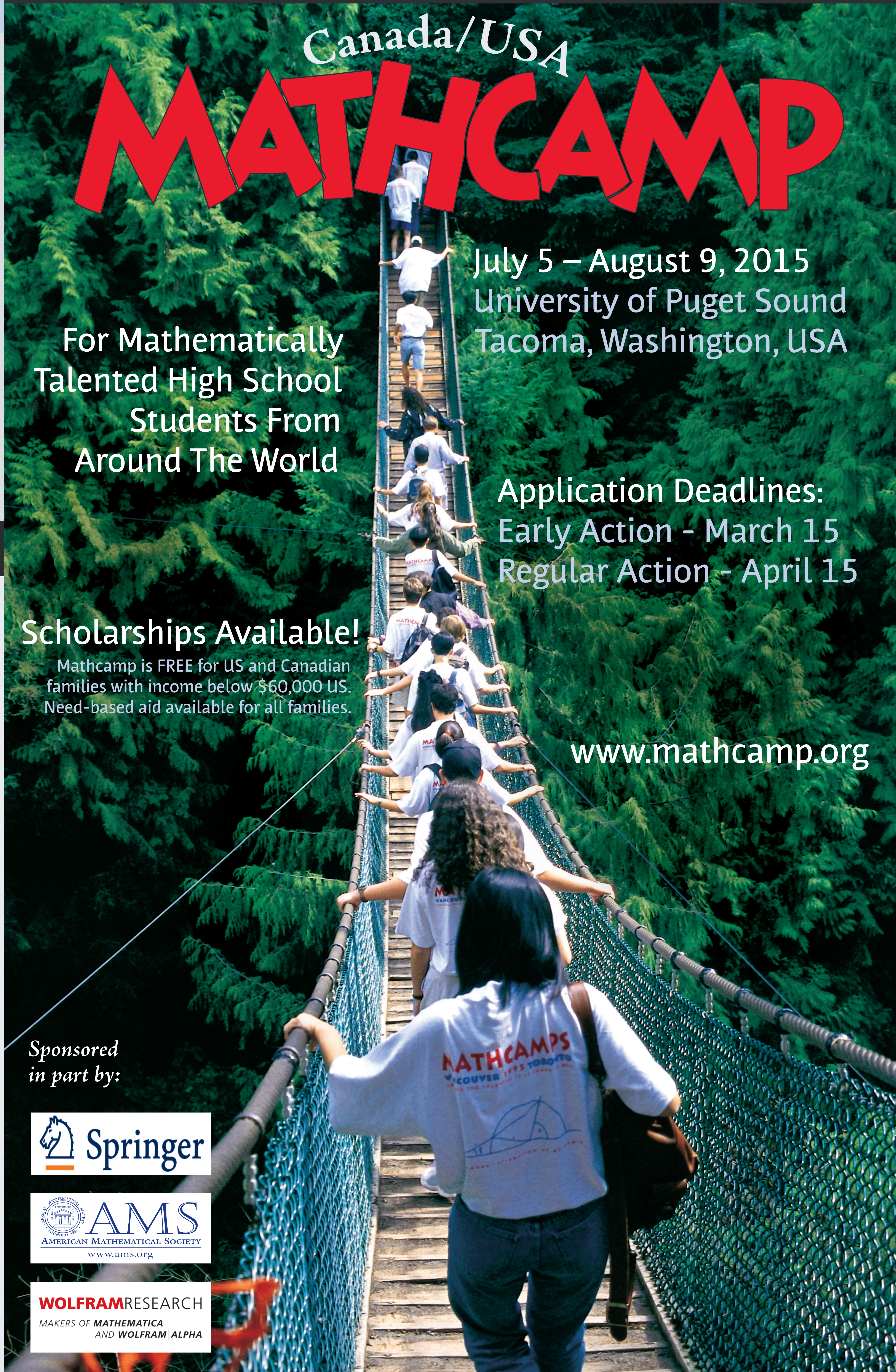
(d) If $n = 2^k$, find a strategy that always forces Alfonso to give out at least $2^{k-1}-1$ bags of gold.

(e) (EXTRA) If $n = 9$, is there a strategy that always forces Alfonso to give out at least 4 bags of gold? (We don't know the answer to this question, but maybe you can figure it out.)

(f) (EXTRA) Can you come up with any other interesting results about this game?

Problem #3 is due to Drake Thomas, MC '14; all other problems by the Mathcamp staff.

The Mathematics Foundation of America invites you to apply to the twenty-third annual



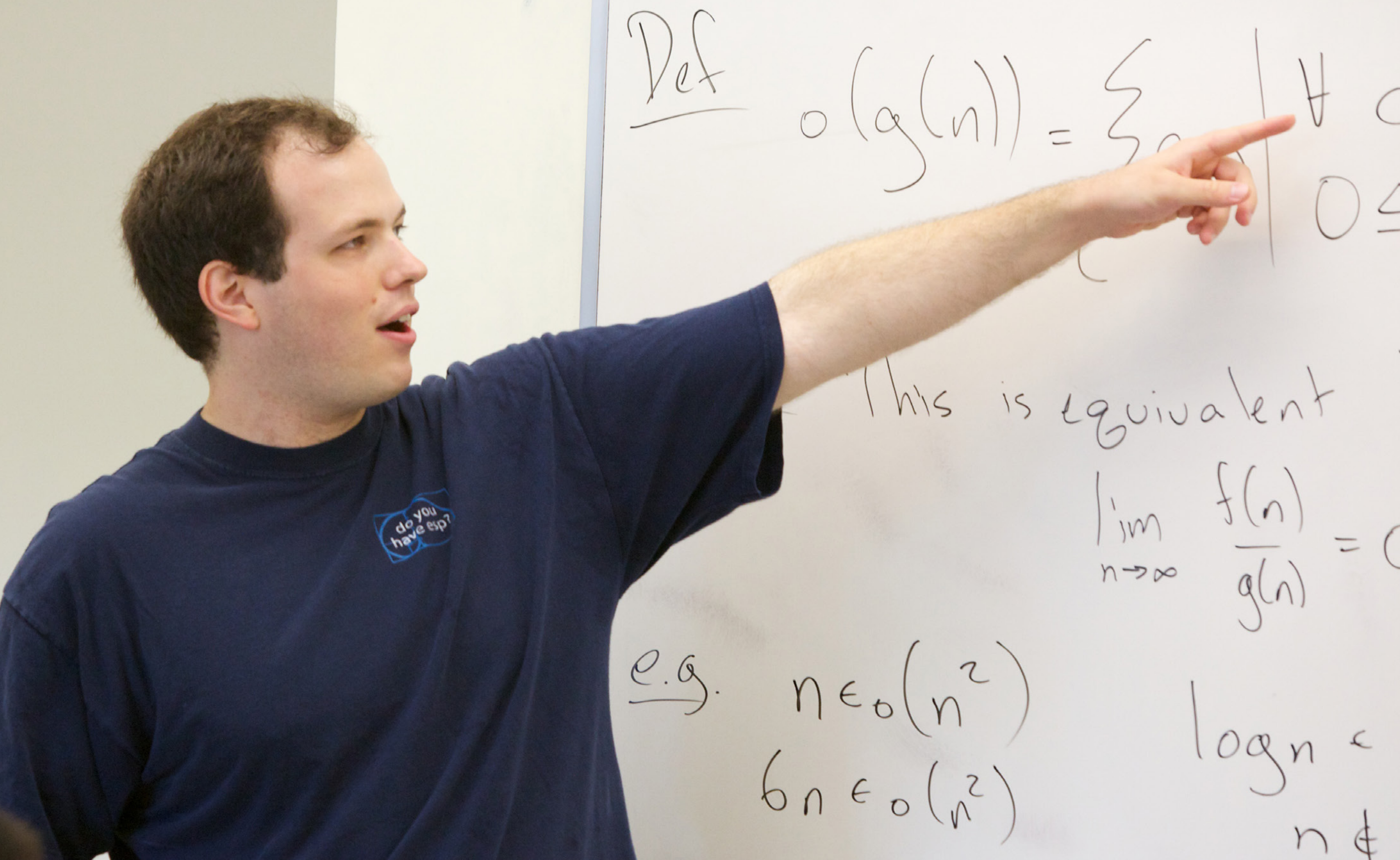
Discover Mathcamp!

"Out of nothing I have created a strange new universe."
– Janos Bolyai, co-discoverer of hyperbolic geometry

Mathcamp is a chance to...

- Live and breathe mathematics: fascinating, deep, difficult, fun, mysterious, abstract, interconnected (and sometimes useful).
- Gain mathematical knowledge, skills and confidence – whether for a possible career in math or science, for math competitions, or just for yourself.
- Set and pursue your own goals: choose your classes, do a project, learn what you want to learn.
- Study with mathematicians who are passionate about their subject, from internationally known researchers to graduate students at the start of their careers, all eager to share their knowledge and enthusiasm.
- Make friends with students from around the world and discover how much fun it is to be around people who think math is cool.

"Mathcamp was the first place where I really understood the beauty and intricacies of abstract mathematics."
– Paul Hlebowitsch (Iowa City, IA, USA)



Academics

A Variety of Choices

The Mathcamp schedule is full of activities at every level, from introductory to the most advanced:

- Courses lasting anywhere from a day to a month
- Lectures and seminars by distinguished visitors
- Math contests and problem-solving sessions
- Hands-on workshops and individual projects

You can learn more at:
http://www.mathcamp.org/academics

The Freedom to Choose

Mathcamp does not have a set curriculum or a list of requirements. We encourage the faculty to teach what they are most passionate about, and we let the students choose what they are interested in learning. With the help of an academic advisor, you will design a program of study that reflects your own interests and goals. You can take any classes you want, and even the number of classes that you attend each day is up to you: you can use your time to review what you've learned, talk to one of your professors, work on problems, do a project, or just take a break. For many students, the freedom to take charge of their own education is one of the aspects of Mathcamp that they value most.

Classes

Course offerings vary from year to year, depending on the interests of the students and faculty. Some of the topics taught in previous years have included:

Discrete Mathematics: Combinatorics • Generating functions • Partitions • Graph theory • Ramsey theory • Finite geometries • Polytopes and Polyhedra • Combinatorial game theory

Algebra and Number Theory: Primes and factorization algorithms • Congruences and quadratic reciprocity • Linear algebra • Groups, rings, and fields • Galois theory • Representation theory • p-adic numbers

Geometry and Topology: Euclidean and non-Euclidean (hyperbolic, spherical, projective) geometries • Geometric transformations • Knot theory • General and algebraic topology • Combinatorial topology • Algebraic geometry

Calculus and Analysis: Fourier analysis • Complex analysis • Real analysis • Measure theory • Dynamical systems • Non-standard analysis • Probability

Computer Science: Cryptography • Algorithms • Complexity • Information theory • Machine Learning

Logic and Foundations: Cardinals and ordinals • Gödel's Incompleteness Theorem • The Banach-Tarski Paradox • Model theory • Category theory

Connections to Science: Relativity and quantum mechanics • Dimensional physics • Voting theory • Bayesian statistics • Neural networks • Mathematical biology • Cognitive science

Discussions: History and philosophy of mathematics • Math education • "How to Give a Math Talk" • College, Graduate School and Beyond

Problem Solving: Proof techniques • Elementary and advanced methods • Contest problems of various levels of difficulty • Weekly "Math Relays" and team competitions

"One cannot compare my ideas of what `I'm interested in math' meant before and after Mathcamp."
– Asaf Reich (Vancouver, BC, Canada)

"There was no pressure: the incentive to learn came from within."
– Keigo Kawaji (Toronto, ON, Canada)

"When I applied, I was really scared to go to a five week camp. What if I were bored or lonely? Having come to Mathcamp, I don't think it's possible to be bored here. I can't imagine a more inclusive, active, or interesting community. I wish it were more than five weeks!"
– Jackie Bredenberg (Bloomfield Hills, MI, USA)

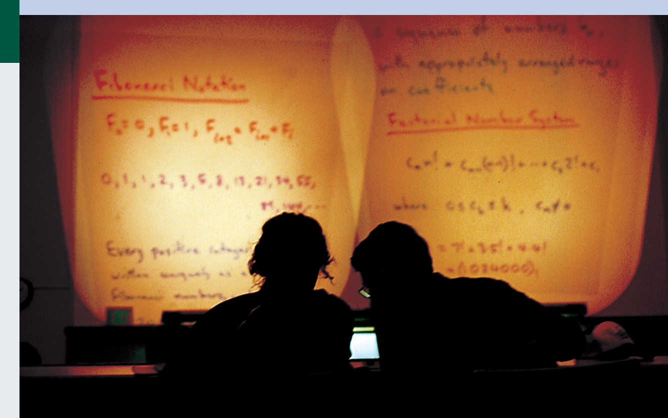
Projects

Every student at Mathcamp is encouraged to do a project, supervised by one of the mentors or faculty. Projects range in scope from creative applications of simple techniques to advanced problems connected to faculty research. Project topics in previous years have included:

- Periodicity of Fibonacci numbers mod n
- NP-Completeness and Latin Squares
- Knight tours on an m-by-n chessboard
- Games on abelian groups
- Constructing the regular 17-gon
- Admissible covers of algebraic curves
- Setless sets in SET, in dimension 4 and beyond
- Computer-generated counterpoint
- The elasticity equation of string
- Light paths in universes with alternate physics
- Playing 20 Questions with a Liar
- Dirichlet's Theorem on Arithmetic Progressions
- Universal Algebra via Category Theory
- The 6.01 Color Theorem
- Teach your own one-hour Mathcamp class!

Spotlight on a Class:
Set Theory as a Foundation for Mathematics

What is a number? Stop and think. Do you know? Can you be sure that numbers even exist (as a mathematical concept)? Believe it or not, this is actually a meaningful question. And fortunately for all of mathematics, the answer to it is "yes". In this class, we're going to create the numbers you know and love – from the naturals up through the reals – out of something even more basic: sets. You might think of sets as "collections of objects", but for us, those "objects" will themselves be sets. We'll start from nothing (the empty set). From that humble beginning, we will build up everything we know about numbers – and more! We'll see how even the most basic properties of numbers can be proved (such as the fact that addition is commutative: $x+y=y+x$). You've probably seen proof by induction before – but have you ever seen a proof that proof by induction works? Once we're done with regular old numbers, we'll use the same techniques to construct infinite (ordinal) numbers. Addition will no longer be commutative, but a certain kind of induction (transfinite induction) will still be valid. By the end of the class, you'll come to see how *everything* we study in mathematics can be expressed using sets. Out of nothing, we will create a strange new universe.



"Mathcamp was definitely the most fun I've ever had."
– Avichal Garg (Cincinatti, OH, USA)

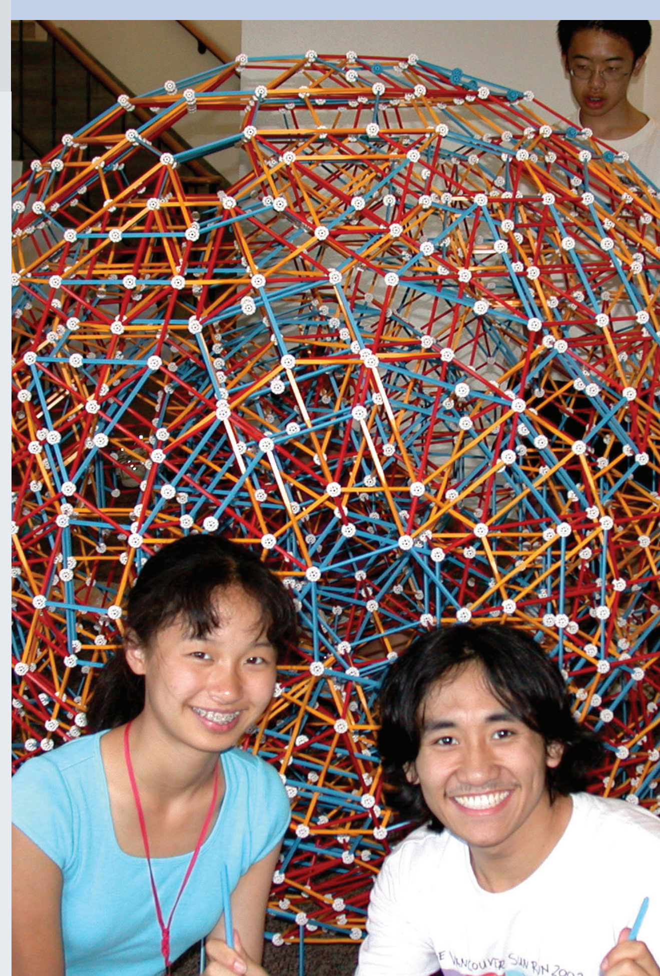
University of Puget Sound

Site of Mathcamp 2015

"Mathcamp isn't really a camp. It's more of a five-week long festival – a congregation of people who celebrate math, enjoy math, learn math and essentially live math. Through it all I've discovered cool theorems that I wouldn't have understood before and cool people I didn't know existed. I've learnt that I actually know close to nothing about the weird and wonderful subject that is mathematics, and that I will probably pursue it for the rest of my life. Math on, Mathcamp!"
– Yongquan Lu (Singapore)

"It's not often that you find a place that is exciting to the mind and liberating to the spirit. Mathcamp is both."
– Greg Burnham (Memphis, TN, USA)

"I've changed so much in my two years here. I think about math in a new, deeper way. I approach problems differently. I've gained perseverence and learned to ask for help without shame and give it with joy."
– Hallie Glickman-Hoch (Brooklyn, NY, USA)



"Go, just go! Trust me!"
– Jian Xu (Toronto, ON, Canada)

People and More

Our Staff

The staff includes Faculty (professors and professionals in math and related fields), Mentors (math graduate students), and JCs (undergraduate students, all of them camp alumni).

Courses at Mathcamp are taught by Faculty and Mentors; each instructor designs her or his own curriculum, picking the course topics freely from among their favorite kinds of math. In addition, we bring several visiting speakers each week who give guest lectures on math and its applications. Outside the classroom, JCs run the non-academic side of camp (from field trips to birthday cakes to frisbee games).

The staff live in the dorms and socialize informally with the students, sharing hiking trips and Scrabble games. Like campers, the staff often return year after year to Mathcamp.

Faculty

Mira Bernstein (MC 1997 – present) is the Executive Director of Mathcamp. Her recent Mathcamp courses have included Combinatorial Game Theory and The Bell Curve.

Mark Krusemeyer (MC 1997 – present) is a Professor at Carleton College. His recent Mathcamp courses have included Galois Theory and Elliptic Functions.

Alfonso Gracia-Saz (MC 2004 – present) is a Lecturer at the University of Toronto. His recent Mathcamp courses have included Point-Set Topology and Voting Theory.

Susan Durst (MC 2008 – present) is a Postdoc at the University of Arizona. Her recent Mathcamp courses have included Inifinite Trees and The Continuum Hypothesis.

Visiting Speakers

Po-Shen Loh (Carnegie Mellon) • Po-Shen Loh studies questions that lie at the intersection of two branches of mathematics: combinatorics (the study of of discrete systems) and probability theory. He is the national lead coach of the USA International Math Olympiad team, and co-founder of expii.com, crowd-sourcing interactive math/science lessons.

Kristin Lauter (Microsoft Research) • Kristin Lauter leads the Cryptography Research Group at Microsoft Research. She loves working on hard math problems which are used in modern cryptosystems, focusing mostly on algorithmic number theory problems. Kristin is currently President of the Association for Women in Mathematics and she is passionate about supporting careers for women and girls in mathematics.

Adam Marcus (Yale University, Crisply) • As co-founder and Chief Scientist of the machine learning startup Crisply, Adam mixes statistics, computer science, and optimization in an attempt to build a "meta-learner" (something that can learn how to learn). In his academic research, he likes looking for the right answers in the wrong places.

"Mathcamp took every limitation I thought I had—social, academic, and personal—and shattered it."
– Andrew Kim (Dover, MA, USA)

Our Students

We never cease to be amazed at the variety of talents and passions our students bring to the program! While everyone at camp shares a love of mathematics, their other interests run the gamut. Each year's camp is a collection of 120 students who are musicians and writers, jugglers and dancers, athletes and actors, artists, board game players, hikers, programmers, students of science and philosophy – all sharing their interests and experiences with each other.

Most of the students at camp come from North America, but many come from overseas. Students have come to camp from Bulgaria, Egypt, India, Japan, Lithuania, Luxembourg, Macedonia, Paraguay, Poland, Romania, Russia, Saudi Arabia, Serbia, Singapore, South Korea, Spain, Switzerland, Tanzania, Thailand, Turkey, and many other places around the globe.

It is a testament to our students' maturity and independence that they can be serious about doing math while still finding so many different ways to have fun. Many camp activities are organized entirely by campers, and students routinely cite each others' company as one of the best aspects of camp.

Beyond Math

Mathematical activities are scheduled for five days a week; whatever math happens on the other two days is purely informal. The weekend is reserved for relaxation and the incredible number of activities that quickly fill the schedule. All of these activities are optional, and students can choose simply to spend time with friends or curl up with a book.

Field trips in the past have included hiking, sea kayaking, whitewater rafting, amusement parks, and museums. Lots of activities happen on campus, too: there are rehearsals for the choir and the contemporary a cappella group, salsa dancing workshops, improv, and bread baking (and subsequent eating). There is an annual team "puzzle hunt" competition, a talent show, and ice cream made with liquid nitrogen. Campers also organize many events themselves—from sports and music to chess and bridge tournaments—and each year, a group of students creates the camp yearbook.

A Note To Parents

Student safety and enjoyment are Mathcamp's first priorities. Students will be housed in secure campus dormitories, with male and female students in designated sections of the same building. Each student is assigned a Mentor or JC as their residential advisor; RAs live on the same hall as their advisees and look out for them on a day-to-day basis. In case of a medical problem, we have a camp nurse at camp or on call, and the hospital is minutes away. Students will have access to college athletic facilities and computers. Every effort will be made to enable students who so desire to attend weekly religious services of their faith. Mathcamp is committed to an atmosphere of mutual tolerance, responsibility, and respect, and we are proud of our past record of creating such an atmosphere.

"Coming to Mathcamp has given me a community with which to interact, not just five weeks a year, but all year round."
– Eric Wofsey (St Louis, MO, USA)