CLASS DESCRIPTIONS — WEEK 1, MATHCAMP 2018

Contents

9:05 Classes

Algebraic Number Theory. (🌶🌶🌶, Shiyue, Tuesday–Saturday)
Every integer can be written uniquely as a product of primes. We all take this for granted, but there are lots of other number systems where unique factorization does not hold. For several decades in the 19th century, mathematicians wrongly thought they had proved Fermat's Last Theorem until they realized this problem. Then Richard Dedekind invented the concept of "ideal". Ideals are not exactly numbers, but in the "ideal" world, unique factorization always works. Unfortunately, this didn't help prove Fermat's Last Theorem, but it generated a huge amount of interesting number theory in its own right.

In this course, we will follow this journey, first finding number systems that don't have unique factorization, then introducing some ring theory and learning to work with ideals, and finally proving Dedekind's great theorem on unique factorization. This is only the first step in the fascinating field of algebraic number theory.

Homework: Recommended

Prerequisites: None; taking group theory at the same time would be useful, but not required.

Related to (but not required for): Intro Number Theory (1/2) (W1); Intro Ring Theory (W2); A Computational Approach to Modular Forms (W2); Commutative Algebra (1/2) (W3); Galois Theory (W3); Algebraic Geometry (2/2) (W4)

Required for: The Class Number (W2)

Crash Course. (🌶, Lara, Tuesday–Saturday)
Mathematicians prove things! So in this class, you'll write lots of proofs. We'll spend a bit of time going over notation you'll need in all your other Mathcamp classes. However for the majority of this class, we'll just answer fun problems and learn how to make our answers water tight, using powerful techniques such as induction and contradiction.

Here are a few problems we're likely to think about together: (1) Show that the set of functions from the positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable. (2) Using the Fermat Numbers $F_n = 2^{2^n} + 1$, prove that there are infinitely many primes. (3) Let a be a nonzero integer. Show that there is no integer n which satisfies the equation $(n - a)^3 + n^3 = (n + a)^3$.

Homework: Required

Prerequisites: None.

Statistical Modeling. (🌶🌶, Sam, Tuesday–Saturday)
This is a weeklong crash course in statistical modelling. We'll cover a general framework for statistical analysis and get our hands dirty analyzing data. We'll mostly focus on linear regression and its extensions, which cover a surprisingly general class of statistical models (including what are often the number 1 and 2 methods on "top ten algorithms for machine learning" lists!). We'll also answer the following questions:

- What was going on with those lines-of-best fit I had to draw in 7th grade science?
- What even is a p-value? Can statisticians agree?
- Can I be more right by agreeing to be wrong on average?
- Does LSD improve your ability to do math (spoiler alert: the answer to this is a resounding no. But we'll work with data from a surreal experiment in the 60s where someone asked this question!)

Fair warning: this is a bona-fide useful class. During homework and lab sessions you'll learn R, a powerful language for statistical analysis. We'll see some statistical theory along the way, but the main goal of this class is for you to leave comfortable with some fundamental and ubiquitous techniques for analyzing data.

Homework: Recommended

Prerequisites: Basic probability: you should know what it means for random variables to be normally distributed and independent, or talk to me!

Related to (but not required for): Probability & Paradoxes (W1); MCMC (W2)

Street Fighting Mathematics. (🌶🌶, *Sanjoy Mahajan*, Tuesday–Saturday)
Street Fighting Mathematics is the art of guessing results and solving problems without doing a proof or an exact calculation. Tools for this include extreme-cases reasoning, dimensional analysis, successive approximation, lumping, and pictorial analysis. And applications include mental calculation, solid geometry, musical intervals, infinite series, and fluid mechanics.

Homework: None

Prerequisites: None

Related to (but not required for): MCMC (W2)

Symmetries and Polynomials. (🌶🌶, Aaron & Apurva, Tuesday–Saturday)
In this course, you'll discover how to solve the cubic and quartic equations. You'll then work out how solving the cubic and quartic equation relates to symmetries of platonic solids and take a peek at Galois theory.

The class will be IBL, specifically, we'll work on problems in class, instead of a traditional lecture format. The basic problem set is 2 chili, but there will be enough optional problems for those who want to make it (much) harder.

Homework: Required

Prerequisites: Familiarity with Group theory will be useful, but not necessary.

Related to (but not required for): Intro Ring Theory (W2); The outer automorphism of S_6 (W2); Representation Theory (1/2) (W3); Galois Theory (W3); Algebraic Geometry (2/2) (W4)

10:10 Classes

Crash Course in Complex Dynamics. (🌶🌶+ *i*🌶, *Scott Kaschner*, Tuesday–Saturday)
This course begins with repeatedly composing functions and, after switching to complex numbers, works its way through fractal geometry and chaos. The internet is awash in pictures of "fractals" that purportedly have something to do with mathematics. We will spend time investigating some of

the algorithms used to generate certain classes of these images. Is it mathematics, artwork, or both? While we can appreciate any of those options, it would be good to know for certain when the pictures are mathematically accurate. Also, the Mandelbrot set will be involved somehow; this is a complex dynamics course after all.

Homework: None

Prerequisites: Calculus (including series)

Related to (but not required for): Chaotic? Good! (W2)

Infinitesimal Calculus. (🌶🌶🌶, Tim!, Tuesday–Saturday)
If you've learned the definition of *continuous function*, you may have learned that a function f is continuous if an infinitely small change in x results in an infinitely small change in $f(x)$. This is a pretty good definition: it's short, and you can picture it on a graph, and you can see the connection to more geometric descriptions ("a function is continuous if you can draw its graph without lifting your pen"). It's also how many of the pioneers of calculus thought about the subject.

But if you take a proof-based calculus class, you might see this definition instead: *A function f is continuous at c if for all $\epsilon > 0$, there is a $\delta > 0$ such that for all x with $|x - c| < \delta$, we have that $|f(x) - f(c)| < \epsilon$.* What an ugly definition! To be sure, it's correct, and is often useful, but nevertheless it's clunky and counterintuitive. Why would any class use it instead of the "infinitely small" definition? The problem is that there is *no such thing* as an infinitely small (or *infinitesimal*) real number.

Most proof-based calculus classes usually throw in the towel on infinitesimals at this point and haul out ϵ and δ instead. But not us. We'll just add some infinitesimal numbers to the real numbers to get the *hyperreal numbers*. And we'll get to have nice definitions like the one at the start of this blurb. We'll go through all the highlights of a calculus class, with proofs that are correct and often much simpler than the standard ones, but which are simultaneously alien and bizarre.

You see, when you start playing with the fundamental building blocks of reality, things can start going totally bananapants. And perhaps we'll come to understand why most calculus classes shy away from infinitesimals.

Homework: Recommended

Prerequisites: Some calculus

Linear Algebra (1/2). (🌶🌶 → 🌶🌶🌶, Mark, Tuesday–Saturday)
You may have heard that linear algebra involves computations with matrices and vectors - and there is some truth to that. But this point of view makes it seem much less interesting than the subject really is; what's exciting about linear algebra is not those computations themselves, but ...

(1) the conceptual ideas behind them, which are elegant and which crop up throughout mathematics, and
(2) the many applications, both inside and outside mathematics.

In this class we'll deal with questions such as: What is the real reason for the addition formulas for sin and cos? What happens to geometric concepts (such as lengths and angles) if you're not in the plane or 3-space, but in higher dimensions? What does "dimension" even mean, and if you're inside a space, how can you tell what its dimension is? What does rotating a vector, say around the origin, have in common with taking the derivative of a function? What happens to areas (in the plane), volumes (in 3-space), etc. when we carry out a linear change of coordinates? If after a sunny day the next day has an 80% probability of being sunny and a 20% probability of being rainy, while after a rainy day the next day has a 60% probability of being sunny and a 40% probability of being rainy, and if today is sunny, how can you (without taking 365 increasingly painful steps of computation) find the probability that it will be sunny exactly one year from now? If you are given the equation $8x^2 + 6xy + y^2 = 19$, how can you quickly tell whether this represents an ellipse, a hyperbola, or a parabola, and how can you then (without technology) get an accurate sketch of the curve? How do astronomers know the chemical composition of distant stars? (We may not get to all these things in the two weeks, but we should cover most of them at least to some extent.)

Homework: Recommended

Prerequisites: Although the blurb refers to taking a derivative, youll be able to get by if you dont know what that means. If you have no previous exposure to abstract concepts, you should at least take the Mathcamp Crash Course at the same time.

Required for: Flag Algebras (W2); Topological Tverberg's theorem (W2); A Computational Approach to Modular Forms (W2); The outer automorphism of S_6 (W2); Linear Programs & Convex Optimization (W3); Representation Theory (1/2) (W3); Spectral Graph Theory (W3); Machine Learning (No Neural Nets) (W4); Cohomology via Sheaves (W4); Rational Points on Elliptic Curves (W4)

Machine Geometry; or, Area and Coarea. (🌶🌶🌶, Misha, Tuesday–Saturday)
This class is inspired by, but not strictly about, computer algorithms for solving geometry problems. We avoid ugly coordinates, but also remain skeptical of proofs that rely on cleverly spotting the right similar triangle or cyclic quadrilateral.

We will begin with the area method, and use it to prove theorems in affine geometry: geometry where we can't measure distances or angles.

We like distances and angles, though. To be able to handle those, we will define a notion of coarea, which relates to area as cosine relates to sine.

Homework: Recommended

Prerequisites: None

Related to (but not required for): How curved is a Potato? (W2); Axiomatic Geometry (W3)

Big Numbers. (🌶🌶, Linus & Pesto, Tuesday–Saturday)
You have 60 seconds to write down the biggest number you can. What do you do? This class is about different ways to get very large numbers:

- From addition to multiplication to exponentiation to power towers and beyond: the method of "Do "Do "Do it again" again" again"
- Structure in randomness: Ramsey Theory & numbers like the famous Graham's Number
- Why can't I write "The largest number definable in less than a thousand letters, plus one?"
- Busy Beaver: one way you *can* write basically that.

Homework: Optional

Prerequisites: None

11:15 Classes

Calculus on Graphs. (🌶🌶, J-Lo, Tuesday–Saturday)
What would happen if you took calculus (the study of how functions change over time) but took out all the continuous stuff? What if instead of the real numbers, you were to input some discrete quantity, like the integers, or the vertices of a graph? Though the computations often become simpler than in regular calculus, we end up in a world with many analogous properties and similar-looking theorems!

In this class, small groups will work together to solve problems that explore this world of functions defined on graphs, and eventually discover what these functions can tell us about the shape of the graph itself.

Homework: Recommended

Prerequisites: None (in particular, calculus is not required)

Related to (but not required for): Intro Graph Theory (W1); Guess Who? (W2)

DIY Hyperbolic Geometry. (🌶🌶, *Katie Mann*, Tuesday–Saturday)
If you've ever done geometry on the surface of a sphere, you'll know that it's a wacky place: what should be "parallel" lines always eventually intersect, the area of a disc is less than πr^2, and when you try to flatten a piece of sphere (perhaps you've tried this with a piece of orange peel), you end up tearing it.

In this class we'll explore an even wilder place to do geometry, the hyperbolic plane. This space is the opposite of the sphere: you can draw both parallel and non-parallel lines that never intersect, the area of a disc is way larger than you expect, and when you try to flatten a piece of it, you are forced to wrinkle it up. You might think you've never seen such a place before, but *actually* hyperbolic is the most common kind of geometry.

This class is called "DIY" for a reason: I'll do some talking, but you should also expect to make and experiment with paper models, attempt to build a hyperbolic soccer ball, and discover the laws of the hyperbolic universe for yourself.

Homework: None

Prerequisites: None

Related to (but not required for): Triangulations and Flip-Graphs (W1); How curved is a Potato? (W2); Metric Space Topology (W2); A Computational Approach to Modular Forms (W2); Low-Dimensional Zoology (W3); Differential Topology (W4)

Game Theory (The Economic Variety). (🌶, Ben, Tuesday–Saturday)
In economics, political science, and even biology, game theory is a useful tool for making predictions. This class will discuss a few of the ideas of game theory, such as: what is a Nash Equilibrium, why do we care about them, and how do we find them? We will also talk about how to formalize our intuition for analyzing games similar to chess or go, where players alternate moves, and what kinds of predictions we end up with here. We'll continue by looking at "repeated games," where players play the same game with each other over and over. Finally, we'll discuss the Folk Theorem, which demonstrates a fascinating difference between analyzing finitely repeated games and infinitely repeated ones.

Although this course will be mostly lecture-based, there will also be several opportunities for discussion, to give a chance to cooperatively work with the concepts of game theory.

Homework: Recommended

Prerequisites: None

Related to (but not required for): Mathematics of Democracy (W4)

Group Theory. (🌶🌶🌶, Mira, Tuesday–Saturday)
Abstract algebra studies how mathematical objects interact and combine to form new objects. For example, numbers combine via addition or multiplication (among other things); functions combine via composition; individual moves on a Rubik's cube combine into more complicated patterns; knots combine by intertwining. Abstract algebra is what happens when you don't care about the objects themselves, but only about the structure of their interaction.

Groups theory lies at the heart of abstract algebra. It examines a type of interaction that occurs over and over in many mathematical contexts: a binary, associative operation with an identity and inverses. (Don't worry if you don't know what that means – we'll explain.) The general results you prove in group theory can be applied to geometry, number theory, combinatorics, topology, physics – basically everywhere! That's why this class is a prerequisite for so many other classes at Mathcamp.

In this course we will cover the basic definitions of group theory, Lagrange's Theorem, homomorphisms, quotient groups, the First Isomorphism Theorem, and a little bit on symmetries and permutation groups. If you've seen most of these topics before, no need to take this class. If you haven't, come join us for a first foray into the beautiful realm of abstract algebra.

Homework: Required

Prerequisites: None.

Related to (but not required for): Trees! (W3); Public-Key Cryptography (W4); Representation Theory (2/2) (W4); Visualizing Groups (W4); Rational Points on Elliptic Curves (W4)

Required for: The outer automorphism of S_6 (W2); Axiomatic Music Theory (W3); Representation Theory (1/2) (W3); Galois Theory (W3)

Stupid Games on Uncountable Sets. (🌶🌶🌶🌶, Susan, Tuesday–Saturday)
Let's play a game. I'll pick a countable ordinal number, then you pick a bigger one, and then I'll pick one that's even bigger. We'll continue this for infinitely many turns, and when we're done we'll check to see who's won. Sound like fun?

As it turns out, these "games" can be a powerful tool for studying important ideas in set theory. In this class, we will learn about the ordinal numbers, clubs, and stationary sets. We will prove the existence of a stationary set which is also co-stationary, and see how this results in a game which has a clear winner and loser, but no winning strategy.

Homework: Recommended

Prerequisites: None

Required for: The Continuum Hypothesis (1/2) (W3)

1:10 Classes

A Mathematician's Perspective on the World. (🌶, *Po-Shen Loh*, Saturday)
"When will we ever use this?" Over the past few years, the speaker has been working to bridge the gap between mathematics and real life, to boost public interest in mathematics. In this talk, he will share some of his mathematical insights on the real world, accumulated from over a year of mathematical introspection on topics of common interest. Some have appeared in features of the New York Times, Wall Street Journal, and FiveThirtyEight (part of ESPN). He will also share his experience in connecting with the general public.

Homework: None

Prerequisites: None.

Intro Graph Theory. (🌶🌶, Mia, Tuesday–Friday)
A new island has been discovered in the Arctic Ocean! While the geographers are arguing over how to divide the island, the cartographers begin to wonder about the map: how many colors are needed to color the countries so that any two countries that share a border get different colors? The Four Color Theorem says just four. However, it took over 100 years and a computer program that checked 1,936 different cases to prove this theorem. In this class, we will won't prove the Four Color Theorem, but we will prove the Five Color Theorem.

Now, let's make this harder. Suppose the countries decide that they have *non-negotiable* color preferences. For instance, the country Zudral demands to be eggplant or magenta. And the country Scaecia refuses to be anything but light blue, sky blue, or cornflower blue. Given that each country now has a list of allowable colors, how does thischange the cartographers' ability to color the map?

Rather surprisingly, the answer is barely. So long as each country has a list of at least 5 colors, the map can be properly colored! That means an analog of the Five Color Theorem still holds. But does the Four Color Theorem still hold? In this class, we will prove some fascinating results in graph theory and eventually answer this question.

Homework: Recommended

Prerequisites: None.

Related to (but not required for): Calculus on Graphs (W1); de Bruijn Sequences (W2); Trees! (W3); Ramsey Theory (W4); Graph Minors (WNone)

Required for: Flag Algebras (W2); Conflict-Free Graph Coloring (W3); Max Flow Min Cut (W3); Spectral Graph Theory (W3)

Intro Number Theory (1/2). (🌶🌶, Mark, Tuesday–Friday)
How do you find $\gcd(a, b)$ for two large integers a and b without having to factor them? Which integers are the sum of two (or the sum of three, or the sum of four) perfect squares? What postages can you get (and not get) if you have only 8 cent and 17 cent stamps available? How does the RSA algorithm (used for such things as sending confidential information, such as your credit card number, over the Internet) work? Besides the answers to such questions, number theory offers insight into many beautiful and subtle properties of our old friends, the integers. For thousands of years professional and amateur mathematicians have been fascinated by the subject (by the way, some of the amateurs, such as the 17th century lawyer Fermat and the modern-day theoretical physicist Dyson, are not to be underestimated!) and chances are that you, too, will enjoy it quite a bit.

Homework: Recommended

Prerequisites: Modular arithmetic (which I can catch you up on, if necessary)

Related to (but not required for): Algebraic Number Theory (W1); Public-Key Cryptography (W4); Rational Points on Elliptic Curves (W4)

Probability & Paradoxes. (🌶, Larsen, Tuesday–Friday)
"This is a very *good* conjecture. It just happens to be false." –G.H.S.

Quick, think of a random number! But what does "random" actually mean? A subtle mathematical theory of *probability spaces* underlies concepts of randomness that may be familiar to you, such as correlation and independence. We'll define these concepts in rigorous ways and use them to prove some conjectures — but some of our conjectures might be false. In probability, many seemingly-rigorous lines of reasoning can lead to contradictory results, or "paradoxes". The Monty Hall problem is a famous example of such a paradox, since an incorrect answer can be very convincing. Fortunately, flawed logic can sometimes be just as interesting as correct logic, so in this class we will study both.

Homework: Optional

Prerequisites: None

Related to (but not required for): Statistical Modeling (W1)

Triangulations and Flip-Graphs. (🌶🌶, Viv, Tuesday–Friday)
Let's say I have two triangulations of a polygon and I want to have some way of getting from one to the other. I can do what's called a diagonal flip, where I remove one diagonal, look at the quadrilateral I've just made, and fill in the other diagonal of that quadrilateral. Can I get from my first triangulation to my second one using only these flips? How many flips will I need? And, most importantly, what does any of this have to do with hyperbolic geometry? We will explore the answers to all of these questions!

Homework: Recommended

Prerequisites: None!

Related to (but not required for): DIY Hyperbolic Geometry (W1); Spectral Graph Theory (W3)

What *should* integration be? (🌶🌶🌶🌶, *Steve Schweber*, Tuesday–Friday)
We know how to find the area under the graph of a "reasonable" function: first consider approximating the area as a bunch of thin vertical rectangular strips, and now just compute the limit as those strips get thinner and thinner (= as our approximation to the area gets more and more precise). For reasonable functions, this limit exists and tells us the area under the graph. This is the **Riemann integral**.

But what about *unreasonable* functions? Consider, for example, the function D given by setting $D(x) = 0$ if x is irrational and $D(x) = 1$ if x is rational. If you try to graph D, you'll quickly see why it's not integrable via Riemann sums. However, there is a heuristic argument (which we'll see in class) that the area under D should in fact be well defined (and equal 0). So we ask: is there a method of integration more general than the Riemann integral, which will let us tackle weird functions like D?

Similarly, the Fundamental Theorem of Calculus tells us that a wide class of antiderivatives can be given by Riemann integrals; however, it turns out that there are functions which have antiderivatives but which are not Riemann integrable. So again, we can ask: is there a method of integration which will let us find antiderivatives of a broader class of functions than the Riemann integral?

It turns out that the answer is *yes*. In this class we'll dive into this strange world. We'll focus on presenting the fundamental ideas behind generalizing the Riemann integral, with a specific focus on two generalizations: the **Lebesgue integral** and the **gauge integral** (or *Henstock-Kurzweil integral*).

Homework: Recommended

Prerequisites: Calculus - specifically, comfort with integrals, Riemann sums, and the epsilon-delta definition of limit.

Related to (but not required for): Convergence Issues; or: Monsters in Real Analysis (W3)

Colloquium

A hex on your fixed points! (*Katie Mann*, Tuesday)
This talk is about two of my favorite things:

(1) board games, and
(2) topology.

The board games part is about Hex, a 2-player game that uses a hexagonal grid. It's easy to learn and to play, but hard (at least for a human!) to figure out a winning strategy. The topology part is a famous theorem of Brouwer that says every continuous map from a disk to itself has a fixed point. Or, as I like to put it: if you stir a bowl of soup, some molecule in the soup always ends up in exactly the same place it started. Brouwer's theorem has lots of deep and beautiful proofs and consequences. But we're going to prove it using Hex.

Dynamical Limits: Sometimes a Picture is Worth Zero Words. (*Scott Kaschner*, Wednesday)
A dynamical system is a mathematical system that evolves over time. In this talk, I will give the history and current status of a project involving a sequence of dynamical systems; it's literally a mathematical system that evolves over time evolving over time. There will be a lot of fractal geometry and, despite my efforts to avoid both, some trigonometry and C++ code. Dynamics researchers are often scientifically inspired by computer-based experimental research; this talk will illustrate how non-directed experimentation in mathematics can lead to interesting (and proved!) mathematical results.

Two equals one: Street-fighting mathematics and science for better teaching and thinking. (*Sanjoy Mahajan*, Thursday)
With traditional science and mathematics teaching, students struggle with fundamental concepts. For example, they cannot reason with graphs and have no feel for physical magnitudes. Their instincts

remain Aristotelian: In their gut, they believe that force is proportional to velocity. With such handicaps in intuition and reasoning, students can learn only by rote. I'll describe these difficulties using mathematical and physical examples, and discuss how street-fighting mathematics and science—the art of approximation—can improve our thinking and teaching, the better to handle the complexity of the world.

The Most Beautiful Equation in Math. (*Po-Shen Loh*, Friday)
What is e? What is π? What is i? What is 1? What is 0? What do they all have to do with each other, and why?

Visitor Bios

Steve Schweber. Hi, I'm Steve (at least, when I'm at camp)! I was a mathcamper for two years, and this is my seventh summer working in some staff-flavored capacity. My main research interest is logic; specifically, I like mathematical logic as a vehicle for discovering terrible counterexamples, cooking up weird alternate mathematicses(?), and generally making the grown-up mathematicians annoyed.

This summer I'm teaching some classes that are sort of about the foundations of geometry: one is about methods of integration more *general* than the usual Riemann integral, and the other is really about ways of measuring the size of 3D shapes that is more *restrictive* than just finding their volume the usual way.

Katie Mann. I study groups acting on manifolds. In other words, I ask how the topology or shape of a space influences the algebraic structure of its group of symmetries or transformations. And then I ask how both of these relate to the dynamics or long-term behavior of transformations: can they be chaotic? Very stable? What do particularly nice examples look like? I like these kinds of questions because they usually require some visual intuition, drawing pictures, and doodling, and on top of that I get to dabble in multiple areas of math (algebra, geometry, topology, dynamics) all at the same time.

Scott Kaschner. My main research interest is dynamical systems. These are mathematical or physical systems that evolve over time. While that description suggests this evolution is continuous (as in differential equations), this is not always the case. Most of my work is in discrete dynamics, where the time steps are repeated applications of a function or mapping to a set.

This field of mathematics has interesting relationships with topology and is one of the driving forces behind the development and study of fractal geometry and chaos. I plan on incorporating all of these ideas into my courses at Mathcamp.

Po-Shen Loh studies questions that lie at the intersection of two branches of mathematics: combinatorics (the study of of discrete systems) and probability theory. He is also passionate about developing mathematical talent at all levels, from coaching the USA International Math Olympiad team, to speaking at math enthusiast gatherings around the world, to crowd-sourcing interactive math/science lessons by all, for all, at `expii.com`.

Sanjoy Mahajan. Sanjoy Mahajan first taught at Mathcamp in 1999. He teaches mathematics, physics, and engineering at Olin College of Engineering where he is interested in improving physics education and in the art of approximation and street-fighting mathematics. He enjoys learning Bach and Handel pieces on the piano, reading, cooking, and languages (one reason that he speaks German to his daughters).

Po-Shen Loh. Po-Shen Loh is a math enthusiast and evangelist. He is the national coach of the USA International Mathematical Olympiad team, a math professor at Carnegie Mellon University, and the founder of expii.com, an educational technology platform which delivers free personalized learning on every smart-phone. His math research considers a variety of questions that lie at the intersection of combinatorics (the study of discrete systems), probability theory, and computer science.