

CLASS DESCRIPTIONS—WEEK 1, MATHCAMP 2017

ADDITIONAL BLURBS

The Pseudoarc. (👤👤👤👤 Steve, Saturday)

Counterexamples are the best! We write down a totally reasonable statement—like “You can’t cut a ball into pieces, rearrange them, and get two balls of the same size as the original”—and then break them with weird ideas. This class is about one particular counterexample: the *pseudoarc*.

Here’s a basic fact about lines: I can cut a line into two smaller lines. A reasonable guess—written a bit informally—is that any “line-like” shape can be cut into two smaller “line-like” shapes. It turns out this is wildly false: the pseudoarc is a kind of lineish thing which can’t be cut into two smaller pieces. The fact that such a weird beast exists at all is surprising; even more surprisingly, in a precise sense *most* shapes are like the pseudoarc!

In this class we’ll define the pseudoarc and sketch its basic properties and the proofs of these properties. Do you like weird shapes of doom? Come to this class!

Homework: Recommended

Prerequisites: Metric spaces

Using Quaternions to Describe Symmetries of Platonic Solids. (👤👤 David Morrison, Saturday)

Have you encountered Euler’s amazing formula $e^{i\theta} = \cos(\theta) + i\sin(\theta)$? Thanks to this formula, rotations in the plane can be described by means of multiplication of complex numbers. It is not hard to go from there to a description of symmetries of regular polygons.

In this course, I will introduce you to quaternions (which you do not need to know about in advance) and explain how they model rotations in three-dimensional space. The multiplication of quaternions plays an important role.

I will also tell you, or if possible extract from the collective knowledge of the students, what the Platonic solids are.

Your project for the rest of the class then becomes to figure out what set of quaternions describes the symmetries, for each of the Platonic solids. We’ll divide into groups for this, with each group tackling the problem for a different solid. The groups will need to stay in contact because progress by one group could be useful for the others.

Prerequisites:

Homework: Optional. We probably won’t finish the project in class, but some of you will enjoy finishing it after class, either individually or in groups.

Prerequisites: Familiarity with complex numbers, including Euler’s formula mentioned above.