



2012 Application Packet

This application packet contains:

- Personal & Contact Information Form
- School & Mathematics Background Form
- Personal Interest Statement Instructions
- The Mathcamp 2012 Qualifying Quiz
- Teacher Recommendation Instructions
- Personal Recommendation Instructions
- (Optional) Financial Aid Application

Please complete all sections of the application for admission (and the scholarship application, if you would like to be considered for need-based financial aid). In addition, please include the US\$20 application fee (check or money order made out to “Mathematics Foundation of America” or a note signed by your parent or guardian explaining that your family cannot afford it. Note that the application fee is waived for online applicants! We strongly encourage you to apply online if you can; the application is available at <http://www.mathcamp.org/apply>).

To apply by mail, submit your application materials in a single envelope to:
Mathcamp 2012 Admissions
129 Hancock Street
Cambridge, MA 02139 USA

All applications **received** (not postmarked!) by **April 25, 2012** will receive equal consideration.

Please do not use certified mail; our office is not always staffed during business hours and we cannot guarantee availability to sign for packages. We will email you when your application has been received.

Thank you for applying to Mathcamp, and good luck!

MATHCAMP 2012: PERSONAL AND CONTACT INFORMATION

First Name: _____ Last Name: _____

Nickname by which you prefer to be called (if different): _____

Date of birth (mm/dd/yy): _____ (must be between August 1993 and July 1999)

Mailing Address: _____

City: _____ State/Province: _____

ZIP/Postal Code: _____ Country: _____

Are you a US Citizen or Permanent Resident? Yes No

If not, what is your country of citizenship? _____

E-mail: _____

Please make sure your email address is legible! Your admission decision will be emailed to this address.

Phone: (_____) _____ I check my e-mail at least twice a week.

How did you hear about Mathcamp? _____

Have you applied to Mathcamp before? If so, in which year(s)? _____

If you plan to fly to Mathcamp, what airport will you most likely be flying from? _____

(We can help participants coordinate flights with others coming from the same place.)

If accepted, I authorize Mathcamp to give out my email address to other campers (e.g. future roommates, or those who live near me, for the purpose coordinating travel to Mathcamp).

If accepted to Mathcamp, will you be able to attend the full five weeks? Yes No

Note: all planned absences from the camp must be discussed with us in advance. Ordinarily, we do not want students to miss more than a few days of camp. Exceptions may be made for conflicts with other summer programs, but students may still miss no more than a week. Only in very special circumstances (e.g. attendance at the International Math Olympiad) will longer absences be allowed. For all absences other than those due to medical or family emergencies, the full camp fee will be charged.

(Optional) Gender: _____ (Optional) Race/Ethnicity: _____

I declare that my solutions to the Qualifying Quiz are my own work. I did not receive any form of assistance from other people, and I have referenced every instance where I looked something up in a book or on the Web.

Signature: _____ Date: _____

MATHCAMP 2012: SCHOOL AND MATHEMATICS BACKGROUND

Name of School: _____ Current grade: _____

High School Curriculum

Which of the following have you studied?

- | | | | |
|---|-------------------------------------|-----------------------------------|--------------------------------|
| Algebra: | <input type="checkbox"/> Not at all | <input type="checkbox"/> A little | <input type="checkbox"/> A lot |
| Geometry: | <input type="checkbox"/> Not at all | <input type="checkbox"/> A little | <input type="checkbox"/> A lot |
| Trigonometry: | <input type="checkbox"/> Not at all | <input type="checkbox"/> A little | <input type="checkbox"/> A lot |
| Exponential and Logarithmic Functions: | <input type="checkbox"/> Not at all | <input type="checkbox"/> A little | <input type="checkbox"/> A lot |
| Differential Calculus (in the US: Calc AB): | <input type="checkbox"/> Not at all | <input type="checkbox"/> A little | <input type="checkbox"/> A lot |
| Integral Calculus (in the US: Calc BC): | <input type="checkbox"/> Not at all | <input type="checkbox"/> A little | <input type="checkbox"/> A lot |

Does your school offer AP Calculus BC, IB Mathematics Higher Level, dual enrollment integral calculus, or an equivalent (both differential and integral calculus)? Yes No

Does your school offer post-calculus math courses? (E.g.: Multivariable calculus, Differential Equations, Linear Algebra. Do not include AP Statistics.) Yes No

Beyond the High-School Curriculum

Does your school have a math team/club? Yes No

Do you attend any extracurricular math activities outside of school? Yes No

If you use the Art of Problem Solving website, what is your username? _____

If you have done any math outside of school (e.g., college courses, online courses, math circles, other summer programs), please tell us about it and briefly describe the material covered.

Math competitions

US & Canadian Students Only: Does your school offer the AMC contest? Yes No

Canadian Students Only: Does your school offer the CEMC contests? Yes No
(The CEMC contests include Fryer, Galois, Hypatia, and COMC.)

If you have participated in any math competitions, please include scores and awards, if any.

MATHCAMP 2012: PERSONAL STATEMENT

Please write a brief personal statement about your interest in math and why you want to come to Mathcamp. Some examples of things you could talk about:

- What would you expect to gain from a summer at Mathcamp?
- What do you like about math?
- Are there specific areas or kinds of math that you enjoy?
- What particularly interesting mathematical idea have you met recently?
- Which of the problems on our quiz did you enjoy most?
- What kinds of mathematical activities (either alone or with other people) are you pursuing at home?

Mathcamp 2012 Qualifying Quiz

Instructions

We call it a quiz, but it's really a challenge: a chance for you to show us how you approach new problems and new concepts in mathematics. What matters to us are not only your final results, but your reasoning. Correct answers on their own will count for very little: you have to justify all your assertions and *prove* to us that your solution is correct. (For some tips on writing proofs, see www.mathcamp.org/proofs.) Sometimes it may take a while to find the right way of approaching a problem. Be patient: there is no time limit on this quiz.

Each problem starts out easier and gets harder; there's a good chance that the first part of each problem will be accessible to you, so *do* try every problem! None of the problems require a computer; you are welcome to use one if you'd like, but first read a word of warning at www.mathcamp.org/computers.

We don't expect every applicant to solve every problem: in the past, we have sometimes admitted people who could do only half of them, occasionally even fewer. However, don't just solve four or five problems and declare yourself done! The more problems you attempt, the better your chances. We strongly recommend that you try all the problems and send us the results of your efforts: partial solutions, conjectures, methods — everything counts.

If you need clarification on a problem, please email quiz12@mathcamp.org. *You may not consult or get help from anyone else.* You can use books or the Web to look up definitions, formulas, or standard techniques, but any information obtained in this way must be clearly referenced in your solution. Please do not try to look for the problems themselves: we want to see how well you can do math, not how well you can use Google! Any deviation from these rules will be considered plagiarism and may disqualify you from attending Mathcamp.

Good luck and have fun!

The Problems

1. A frog jumps along the number line. It starts at 0 and every second it jumps n units to the right (the same positive integer n each time). After one second, you decide that you want to catch the frog. It's dark, you can't see the frog, and you don't know what n is. (For all you know, it might be a super-frog, so n could be arbitrarily large.) However, at any given second, you are allowed to choose an integer and search there. If the frog is on that integer, you'll catch it; if not, you'll have to try again.
 - (a) Devise a strategy that will eventually catch the frog. (You'll need to explain which integer you plan to check at each second.)
 - (b) Now suppose the frog is allowed to start by going either to the left or to the right; once it chooses a direction, it always jumps n units in that direction. Can you devise a strategy for catching it if you don't know which way it's going, and don't know what n is?
 - (c) What if the conditions in part (b) hold, and you also don't know which integer point the frog started at? (After you have worked on this problem for a while, it may be useful to read the following article, particularly the section after the proof of Lemma 1: http://www.cut-the-knot.org/do_you_know/numbers.shtml.)
2. Each integer on the number line is colored with exactly one of three possible colors – red, green or blue – according to the following rules:
 - the negative of a red number must be colored blue,
 - the sum of two blue numbers (not necessarily distinct) must be colored red.
 - (a) Show that the negative of a blue number must be colored red and the sum of two red numbers must be colored blue.
 - (b) Determine all possible colorings of the integers that satisfy these rules.

3. Let p be an odd prime. A group of p campers sit around a circle, and are labeled with the integers $1, 2, \dots, p$ in clockwise order. The camper with label 1 yells out the number 1. The camper sitting next to this camper in clockwise order yells out 2. The camper two spots in clockwise order from the camper who yelled out 2 yells out 3. This process continues: the camper seated n spots (in clockwise order) from the camper who yelled out n must yell out $n + 1$. A camper gets a cookie anytime she or he yells out a number.
- Show that there is a camper who never gets a cookie.
 - Of the campers who do get cookies, is there one who at some point has at least ten more cookies than the others?
 - Of the campers who do get cookies, is there one who at some point has at least ten fewer cookies than the others?
4. Let a be a rational number with $0 < a < 1$. A lollipop in the xy -plane with base $(a, 0)$ consists of a line segment from $(a, 0)$ to some point (a, b) with $b > 0$, together with a filled in disc of radius less than b , centered at (a, b) . Determine whether or not it is possible to have a set of lollipops in the xy -plane satisfying both of the following conditions:
- for every rational number a with $0 < a < 1$, there is a lollipop whose base is the point $(a, 0)$,
 - no two lollipops touch or overlap each other.

If such a set of lollipops exists, explain how to construct it. If not, justify why not.

5. A convex body in the plane is a region with positive area such that for any two points in this region, the entire line segment between them also lies within the region. Let P be the perimeter (i.e., boundary) of a convex body in the plane. We will assume throughout this problem that P is *centrally symmetric*: that is, if (a, b) is a point on P , then so is $(-a, -b)$.

For any nonnegative real number k , we define kP to be the subset of the plane obtained by multiplying all the points of P by k in each coordinate. In other words, for each point (a, b) of P , the point (ka, kb) is in kP .

If $(x_1, y_1), (x_2, y_2)$ are two points in the plane, we define the P -distance between them to be the smallest nonnegative real number k such that when the set kP is translated by (x_1, y_1) (i.e., by x_1 units horizontally and by y_1 units vertically), the point (x_2, y_2) lies on it. For example, if P is the square with vertices $(0, 1), (1, 0), (0, -1), (-1, 0)$, then the P -distance between $(3, 5)$ and $(4, 10)$ is 6.

- Let P be the perimeter of a disc of radius 1 centered at the origin. Find a formula for the P -distance between any two points (a, b) and (c, d) in the plane.
- Let P be the perimeter of a rhombus with vertices $(2, 0), (-2, 0), (0, 3), (0, -3)$. Find a formula for the P -distance between any two points (a, b) and (c, d) in the plane.
- In part (a), we took it for granted that a filled-in disc of radius 1 is a convex body. Prove this rigorously, using the definition of convexity given above.
- Suppose P is a convex quadrilateral. What are the possible P -distances between vertices of P ? What about when P is a convex hexagon? (Remember: P must still be centrally symmetric!)
- Let P be the perimeter of some centrally symmetric convex body, and let (a, b) be a point on P . What is the largest possible P -distance from (a, b) to another point on P ? Will (a, b) be at this P -distance from just one other point on P or from multiple other points? (If any of your answers depend on the geometry of P and/or on the choice of (a, b) , explain how.)
- In principle, we could define P -distance even when P doesn't come from a convex body and/or is not centrally symmetric. But it turns out that in both of these cases, the definition is problematic: the resulting quantity doesn't behave in the ways we expect a "distance" to behave. Can you determine what problematic issues arise?

6. An ocean has infinitely many islands. Every island is labeled by one of the integers $\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$, with no two islands having the same label and every integer being the label of some island. Two islands are connected by a bridge if their labels differ by a power of two. For instance, there is a bridge connecting island 7 and island -25 .

We define the *distance* between two islands k_1 and k_2 to be the minimum number of bridges needed to get from k_1 to k_2 . For instance, the distance between the islands 0 and 7 is 2. (You can move from island 0 to island 8, then to island 7; this is the minimum, since you can't go from 0 to 7 using just one bridge.)

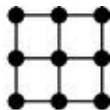
- (a) Show that for any integer $r \geq 1$, you can find two islands in the ocean at distance r from each other.
- (b) An *infinite path* in our ocean consists of an infinite set of islands \mathcal{I} and an infinite set of bridges \mathcal{B} , such that:
- every island in \mathcal{I} is connected to exactly two bridges in \mathcal{B} ,
 - for any two islands in \mathcal{I} , you can get from one to the other using only bridges in \mathcal{B} .

Here is one example of an infinite path: let \mathcal{I} be the set of all odd-numbered islands and let \mathcal{B} be the set of bridges between islands in \mathcal{I} whose labels differ by 2. It is easy to see that \mathcal{I} and \mathcal{B} satisfy both conditions for an infinite path:

- Each island k in \mathcal{I} is connected to exactly two bridges in \mathcal{B} – namely, the bridges leading to islands $k - 2$ and $k + 2$;
- To get from any odd-numbered island to any other using bridges in \mathcal{B} , you start at the smaller number and keep adding 2.

As a warm-up, can you create an infinite path with the *same* \mathcal{I} as in the example above, but with a *different* \mathcal{B} ?

- (c) Is it possible to construct an infinite path in our ocean such that, for any two islands k_1, k_2 in \mathcal{I} , the minimum number of bridges in \mathcal{B} needed to get from k_1 to k_2 is exactly the distance between k_1 and k_2 ? For instance, the infinite path in our example does *not* have this property: it takes two bridges in \mathcal{B} to go from island 1 to island 5 (you have to go via island 3), even though the distance between these two islands is 1 (there is a single bridge not in \mathcal{B} that connects them to each other). If your answer is yes, give an example of sets \mathcal{I} and \mathcal{B} that work. If your answer is no, prove that it can't be done.
- (d) Does there exist a set \mathcal{S} of 9 islands such that:
- the configuration of bridges connecting pairs of islands in \mathcal{S} is exactly as in the picture below (with no additional bridges between any of the islands), and
 - the distance between any two islands in \mathcal{S} equals the minimum number of bridges needed to get from one island to the other via islands in \mathcal{S} ?



What if, instead of a 3×3 grid, we had an $n \times n$ grid: for which n is such a configuration possible?

- (e) Suppose, in a sea far away, we have islands labeled in the same way, with two islands connected by a bridge if their labels differ by a power of 3. Is there a one-to-one correspondence between islands in the ocean and islands in the far-away sea, such that two islands in the ocean are connected by a bridge precisely when the corresponding two islands in the sea are connected by a bridge?

7. Last summer, the graduate students teaching at Mathcamp (we call them “mentors”) arranged themselves into a pyramid with four layers:



Now suppose we generalize this to a pyramid with n layers (n mentors in the bottom row, $n - 1$ mentors in the row above, etc.). Assume that all mentors have weight 1 and that each mentor supports his/her own weight plus half the weight supported by the one or two mentors leaning on him/her. For instance, in our four-layer mentor pyramid, the weights supported by the mentors are

layer 1:		1		
layer 2:		$3/2$	$3/2$	
layer 3:	$7/4$		$5/2$	$7/4$
layer 4:	$15/8$	$25/8$	$25/8$	$15/8$

- (a) Find the weight supported by the mentor at the bottom left corner of a pyramid with n layers.
- (b) Now suppose the pyramid has infinitely many layers. Let $W(k, m)$ be the weight supported by the $(k + 1)$ -th mentor from the left in the $(m + 1)$ -th layer. For instance, $W(0, 0) = 1$, $W(0, 1) = W(1, 1) = 3/2$, etc. Determine a recursive formula satisfied by the function $W(k, m)$, for $k, m \geq 0$.
- (c) Find an explicit formula for $W(k, 2k)$ in terms of k , for $k \geq 0$. If you can't find a fully explicit formula, can you find one that uses summation notation?
- (d) **Extra credit:** What can you say about $W(k, m)$ in general?

Note: If you're not sure what we mean by “explicit” and “recursive” formulas, take a look at <http://www.regentsprep.org/Regents/math/algtrig/ATP3/Recursive.htm>.

Mathcamp 2012: Academic Recommendation Letter

Name of student: _____

Name of recommender: _____

To the recommender:

The above-named student is applying to Canada/USA Mathcamp (<http://www.mathcamp.org>), an intensive summer program that introduces mathematically talented high-school students to the power and variety of advanced mathematics. Mathcamp receives applications from all over the world, and our admission process is extremely selective. We appreciate your help in evaluating this student's application.

The student has listed you as a teacher who knows him or her well. In your recommendation, please comment on the student's creativity, initiative, ability to work independently and with others, as well as his or her mathematical background and academic achievements. Do you feel that a program like Mathcamp is suitable for this student? Are there any special circumstances in the student's background that we should be aware of?

Your recommendation can be submitted in any of the following ways:

- Online, at <http://www.mathcamp.org/recommendations> (preferred)
- By email, to admissions12@mathcamp.org (next best)
- By fax, to (888) 371-4159
- By mail, to Mathcamp 2012 Admissions, 129 Hancock St, Cambridge, MA 02139, USA
- By giving your letter to the student in a sealed envelope, with your signature across the seal.

In order for the student's application to be given full consideration, your recommendation must be received by **April 27, 2012**.

Thank you, once again, for your help in this process and for supporting talented students in mathematics!

The Mathcamp 2012 Admissions Committee

Mathcamp 2012: Personal Recommendation Letter

Name of student: _____

Name of recommender: _____

To the recommender:

The above-named student is applying to Canada/USA Mathcamp (<http://www.mathcamp.org>), an intensive summer program that introduces mathematically talented high-school students to the power and variety of advanced mathematics.

Mathcamp receives applications from all over the world, and our admission process is extremely selective. We are looking for students who are not only good at math, but who will thrive in the atmosphere of freedom and responsibility that characterizes Mathcamp, and who will make a positive contribution to the camp community. We appreciate your help in evaluating this student's application.

The student has listed you as a personal reference. In your letter, please comment on his or her maturity, independence, social and personal qualities. Do you feel that a program like Mathcamp is suitable for this student? Are there any special circumstances in the student's background that we should be aware of?

Your recommendation can be submitted in any of the following ways:

- Online, at <http://www.mathcamp.org/recommendations> (preferred)
- By email, to admissions12@mathcamp.org (next best)
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In order for the student's application to be given full consideration, your recommendation must be received by **April 27, 2012**.

Thank you, once again, for your help in this process!

The Mathcamp 2012 Admissions Committee

MATHCAMP 2012: FINANCIAL AID APPLICATION

Admission to Mathcamp is need-blind. Any financial information you provide will not affect your chances of acceptance. We are deeply committed to enabling every qualified student to attend, regardless of financial circumstances. Please do not let financial considerations prevent you from applying!

If you think that you will need financial assistance to attend Mathcamp, please tell us now rather than later. Scholarship decisions are made immediately after admissions decisions. If you would like to be considered for a scholarship, **ask a parent or guardian to complete the scholarship application below (or submit responses on a separate sheet of paper).**

Annual 2011 family income (all sources): US \$ _____

Please estimate your 2012 family income. If you expect your family income in 2012 to be significantly different from 2011, please explain.

Please list *all* members of your household who are supported by the income above (including the applicant) and their relationships to the applicant. For siblings, please provide ages.

Please explain the cost of schooling (e.g.: private school, college, etc.), if any, for all household members, including the applicant.

Estimate the cost of round-trip travel to Mathcamp for the student: US\$ _____

How much can your family afford to pay for Mathcamp (*including travel costs*)? US\$ _____

On a separate sheet, please describe any special circumstances that you would like us to consider.

Name of parent/guardian filling out this form: _____

Email address (to contact you if we have further questions):

Relationship to student: _____ Signature: _____