CLASS PROPOSALS—WEEK 5, MATHCAMP 2008

Contents

Read carefully!

The Week 5 schedule is decided by you, the campers! This handout contains the description of 2^6 classes that we could offer in Week 5. Read them carefully and vote on our favourites. Then we will try to produce a schedule that makes you all happy.

In order to vote, go to the usual url and fill out the survey, which consists simply on checking off the classes you would like to see in Week 5. You also have a chance to propose any other regular class that you would like to see continue in Week 5, even if they do not appear in this handout.

The deadline to vote on Week 5 classes is Thursday, July 31st, at 7pm.

You may certainly vote for more classes that you could possibly attend. After all, some of them might conflict and not all of them will make it to the schedule. But vote only for classes that you are willing to attend, *even if they happen on Friday at 9am.* Keep also in mind that voting for all classes has the same effect as voting for none of them.

Unless otherwise stated, Week 5 classes have no homework.

Sets, Scales, and Serialism. (∗∗, Andre Kuney, 1 day)
Have you ever heard music that just sounded *weird*? Most "classical" music written in the past 100 or so years doesn't exactly sound like Mozart, Beethoven, or the Beatles. However, as I will attempt to convince you, there is a method to the madness. Join me on an hour-long whirlwind tour of generated scales, tone rows, pitch classes, hexachords, and atonality. I will show you the methods music theorists use to attack modern music (combinatorics, groups, and more!), demonstrate how they relate to "traditional" music theory, and prove a theorem or two in the process. Prepare to think about music like never before!

Prerequisites: Knowing basic music terms like "major," "minor," "octave" and "scale" would help.

Related to: Math and Music, Group Theory.

The Navier-Stokes Equations. (∗∗∗, Asaf Reich, 2 days)
What do the flow of air around a plane, the waves and currents in the ocean or a pool, and a million dollars have to do with each other? The Navier-Stokes Equations, a system of partial differential equations that describe almost all flow of liquids and gases. Though they have been studied intensively, they're also extremely difficult to analyze - in fact there's a 1 million prize for proving there even exist (unique) smooth solutions for all time. In this class, on the first day we'll introduce the N-S Equations and give some non-rigorous, heuristic derivations of them from physical principles. Come if you'd like to understand a bit about the math that governs things like vortices in water and pretty much everything else, or if you just want to know what all the interest is about. On the second day we'll be doing more rigorous math: it turns out that for 2-dimensions, given some assumptions about the flow, the N-S equations become vastly simpler to analyze. We can can prove neat and useful statements like Bernoulli's Equation, and even cooler, we can actually convert the problem into complex analysis and reduce flows to analytic functions. If we have time we'll use this to derive an explicit formula for the flow of water around a circle.

Prerequisites: For the first day: calculus, some very basic physics like $F = ma$ and pressure. For the second day, complex numbers as well, and although I'll introduce the necessary multivariable and complex analysis, some familiarity with these topics might be helpful.

Related to: Differential Equations and Mathematical Modeling.

Transfinite Induction. (∗∗∗, Eric Wofsey, 1 day)
Consider the intermediate value theorem: if $f : [a, b] \to \mathbb{R}$ is a continuous function and $f(a) \leq d \leq f(b)$, then there is some $c \in [a, b]$ such that $f(c) = d$. Here's one way you might try to prove this theorem. We know that $f(a) \leq d$. If $f(a) = d$, we can set $c = a$ and we're done; otherwise by continuity there is some a_1 such that $a < a_1 \leq b$ and $f(a_1) \leq d$. If $f(a_1) = d$, we can set $c = a_1$ and we're done; otherwise by continuity there is some a_2 such that $a_1 < a_2 \leq b$ and $f(a_2) \leq d$. We can continue this argument by induction. If we never have $f(a_n) = d$, then $f(a_n) < d$ for all n, and we can let $z = \lim a_n$, and again by continuity $f(z) \leq d$. We then can start the argument all over again with z replacing a. If we keep repeating this argument over and over again, we have to eventually reach b, so since $f(b) \geq d$ we must eventually find a value c such that $f(c) = d$.

If you're skeptical of this proof, you have good reason: we never justified how we "eventually reach b" by repeating the induction over and over again. However, it turns out that this proof *does* work if you use a more powerful kind of induction called *transfinite induction.* Basically, transfinite induction allows you to continue an induction argument "over and over again", even after you "finish" the induction (as we did when we ran out of a_n's and had to start over with z). In this class we'll build up the technical machinery required to make transfinite induction (and the proof above) work. While the word "transfinite" may sound scary (and some professional mathematicians think it does!), I hope to convince you that with the necessary machinery, transfinite induction is just as easy and intuitive as ordinary induction.

Prerequisites: Familiarity with $\varepsilon - \delta$ proofs helpful but not required; fearlessness in the face of abstraction.

Related to: Logic, set theory, proof techniques, real analysis.

Algebraic Graph Theory. ($*{*}*$–$*{*}{*}*$, Jacob Steinhardt, 1 day)

Given a graph X, we can associate a matrix A to X, called its *adjacency matrix*, defined by $A_{vw} = 1$ if vertices v and w are adjacent, and $A_{vw} = 0$ otherwise. For example, for the graph on 4 vertices with edges $\{\{1,2\},\{1,4\},\{2,3\},\{2,4\}\}$, the adjacency matrix would be

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

So, given a *combinatorial* object (a graph), we can get from it an *algebraic* object (a matrix). In this class we'll explore how we can use the adjacency matrix together with facts about linear algebra to derive some facts about graphs. I'll start with some basic ideas in algebraic graph theory. After that, I'll give an algebraic proof of problem 6 on this year's USAMO, and then an algebraic proof of another olympiad problem with a perniciously difficult combinatorial proof. If I have time at the end, I'll try to talk about some of the general ideas in algebraic graph theory.

For those interested, here is the statement of the two problems:

- *Problem 1:* Given a graph G, show that it is always possible to partition the vertices into two sets S and T such that each vertex is adjacent to an even number of vertices in the same partition. Furthermore, show that the number of ways to construct such a partition is a power of 2.
- *Problem 2:* Start with an $m \times n$ grid of integers. Every second, simultaneously replace each integer with the sum of the integers in adjacent squares. For what ordered pairs (m, n) will all of the integers eventually be even, no matter what the original integers were?

Prerequisites: Graph theory, linear algebra, and basic facts about groups and fields. (*Note:* we'll be doing linear algebra over $\mathbb{Z}/2\mathbb{Z}$ instead of $\mathbb{C}$, but it works mostly the same. In fact, it is simpler!) I will also be using the fact that the non-zero elements of any finite field form a cyclic group under multiplication. If you understand the previous statement and are comfortable taking it up on faith, then that is enough.

Homework: None required, but I'll give you some interesting problems to think about at the end of the class.

Related to: Graph theory, algebra, problem solving.

Alice's Classes

Postmodern Mathematics. ($*$, Alice & Greg, 1–4 days)

This class will be a discourse on the dominant paradigms of mathematical inquiry. Is mathematics primarily an exercise in semiotics or is the platonic model substantial? What is the nature of mathematical existence? Can we adhere to the axiomatic paradigm, or does the essential lack of completion undermine them at the core? We'll cover the pro-choice vs. pro-ZF culture wars and debate the views of the constructivists. We'll also try to decide if we can really say "Tertium non datur" and follow the principium tertii exclusi. We will eschew the dominant lecture paradigm; the class is discussion based.

Prerequisites: If you were part of the postmodernism activity that's great, but definitely not required.

Homework: There may be short recommended nightly readings.

Alfonso's Classes

The stable marriage problem. ($*$–$**$, Alfonso, 1 day)

N single men and N single women want to pair up and get married. These are their names and preferences:

- Alfonso: Nina, Susan, Marisa, Yvonne, Katya.
- Dan: Katya, Nina, Yvonne, Susan, Marisa.
- Greg: Katya, Yvonne, Susan, Nina, Marisa.
- JR: Marisa, Susan, Yvonne, Katya, Nina.
- Paul: Marisa, Katya, Nina, Yvonne, Susan.

- Katya: Alfonso, JR, Paul, Greg, Dan.
- Marisa: Greg, Dan, Alfonso, Paul, JR.
- Nina: Greg, JR, Paul, Dan, Alfonso.
- Susan: Greg, Paul, Alfonso, Dan, JR.
- Yvonne: Greg, JR, Dan, Paul, Alfonso.

Is it possible to make everybody happy? Obviously not since everybody wants to marry Greg. But is it possible to at least create a *stable* situation? For instance, it is a bad idea for Greg to marry Susan and for Yvonne to marry JR, because then Greg and Yvonne would prefer each other rather than staying with their partners, so they will run away together. How can we at least avoid having a run-away couple? Is there more than one way to do it? What is the *best* way to do it? What if we move to Spain, where Marisa and Nina can ignore Greg and marry each other?

Prerequisites: None.

Voting theory. ($*$, Alfonso, 2 days)

When a large group of people have to make a decision together, bad things can happen. For example, suppose that a group of 10 campers is trying to decide which game to play tonight. Suppose further that 3 of them want to play Mao, and the remaining 7 would prefer to play *any* game they can possibly think of other than Mao. If the remaining 7 are divided between 5 or 6 different games, a strict plurality election system will force them to play Mao, even though a majority of the 10 campers would prefer any other candidate to the winner.

It seems, then, that the plurality election system is unfair. What could we do to make it fair? Which election system are the most fair? What does "fair" mean, anyway? Come to this class and find out. *Warning:* Your faith in democracy may vanish after this class.

Note: This class blurb was shamelessly stolen from former Mathcamp mentor Dave Jensen.

Prerequisites: None.

The feeling of power. (∗, Alfonso, 1 day)

Suppose that you are marooned on a desert island without a calculator. You invent a clever scheme to escape which involves building a rocket ship out of coconut husks, but in order to calculate the right fuel ratio you end up needing to take a square root. Fortunately, you went to this class, so you know a way to take square roots manually to arbitrary decimal places, like long division, without using a calculator. In fact, you'll still be okay even if you need to take a cubic root!

Note: This class blurb and idea were shamelessly stolen from Anti.

Prerequisites: None.

The Pascal triangle mod p. (∗–∗∗, Alfonso, 1–2 days)

Write the first ten rows of the Pascal triangle. Circle the odd numbers. Can you see the pattern? Can you describe the pattern effectively? Can you prove that your description is correct? Now write the first few rows of the Pascal triangle modulo a prime p. Can you describe the pattern *now*? The answer is given by a cute, little, and not very well-known theorem.

Prerequisites: None.

Related to: Number theory.

Nets. (∗∗∗∗, Alfonso, 4 days)

So you know point set topology. You are probably quite familiar with sequences and their properties. Let me tell you something, baby: Sequences ain't enough! Sequences are not capable of fully understanding what topological spaces are about. Here are some problems (there are plenty of mathematicians who get the first two of these wrong!):

- If a sequence has a convergent subsequence, then it has an accumulation point. The converse is not true.
- In a Hausdorff topological space, every sequence has at most one limit. The converse is not true.
- If a function $f : X \to Y$ is continuous, then for every sequence $\{x_n\}$ convergent to x in X, the sequence $\{f(x_n)\}$ converges to $f(x)$ in Y. The converse is not true.
- A compact space does not have to be sequentially compact, and a sequentially compact space does not have to be compact.

How do we fix this? Throw away sequences, and replace them with their natural uncountable counterpart: *nets* (or, if you are Waffle or Anti, replace them with *filters* instead). I will teach you all about nets, how they fix the above problems, and then we will prove the Axiom of Choice.

Prerequisites: You need to be very comfortable with Point Set Topology and understand most of the statements of the four bullet points above. If in doubt, ask me. You also need to be comfortable with the (uncountable) product topology; otherwise the proof of the Axiom of Choice will be a bumpy ride.

Related to: Point set topology.

Computability & Complexity, cont'd. ($***$, Dan, 4 days)

Up until now, we've tried to understand how certain theoretical "abilities" that you can give to a Turing machine (such as nondeterminism) affect its computational power. Now we'll look at two other interesting ways to define variations on computation: probabilistic computation and interactive proof systems.

Probabilistic computation is essentially designed to give a Turing machine the ability to get as input random data. To probabilistically decide a language L, the machine must be correct on any given input w at least $\frac{2}{3}$rds of the time. Thus the machine may not always be correct, but by running it repeatedly you can be certain if $w \in L$ with high probability. Such algorithms are used throughout modern computer science and are extremely important; understanding how this relates to P and NP is a fundamental question.

Interactive proofs, which I hinted at on the last day of class, capture a different, subtle issue: for what questions (i.e. languages) can we design an efficient (polynomial-time) verifier that can, with high probability, verify the answer given by a powerful "prover?" Whereas the notion P vs. NP was about what questions have short proofs, the idea of interactive proofs generalizes this to allow for questions to be asked to the prover.

Prerequisites: Computability & Complexity, or TCS in previous years.

Homework: Recommended.

The Classification of Surfaces. ($**$, Dan, 4 days)

A *surface* is a topological space that looks locally two-dimensional: that is, like a plane, but possibly curved. Examples of surfaces are objects like spheres, tori, two-holed tori, the projective plane, and the Klein bottle.

We're going to study the family of all such objects and see if we can find out which ones are essentially the same. It might be surprising that there would be any structure at all to the idea of "every space that looks locally two-dimensional," since we have no restrictions on how to construct them! However, this is actually a tractable question; by the end of the class, you'll understand what every possible such space is, even if it can't be embedded in three dimenions, so that we can't ever see a fully accurate picture of it.

Prerequisites: None.

Homework: Recommended, but only about 15-30 minutes per night.

Related to: Topology

Smooth Manifolds. ($****$, Dan, 4 days)

A smooth manifold is, in some sense, a space with the least amount of structure necessary to do calculus. Many "nice" spaces are manifolds, and you can do a number of interesting and surprising things once you require this smoothness condition.

We'll investigate manifolds, see how to define them and how to work with them. We might try to pursue the generalization of Stokes' theorem on manifolds, or we might go in a different direction, but either way we'll see some of the surprising things that can happen on abstract spaces whenever the idea of a "tangent vector" still makes sense.

Prerequisites: Calculus (preferably multivariable) and point-set topology.

Dave's Classes

Bernoulli Numbers. (∗∗∗, Dave, 4 days)

Here are three facts:

- The sum of the fourth powers $1^4 + 2^4 + \cdots + (m-1)^4$ is equal to $\frac{m^4}{4} - \frac{m^3}{2} + \frac{m^4}{4}$;
- We have infinite sums $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$ and $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$;
- The special case of Fermat's last theorem $x^7 + y^7 = z^7$ has no integer solutions with x, y, z nonzero.

In this class we'll explain why these three facts — and generalizations of them involving higher powers — are more or less equivalent.

Prerequisites: Number Theory, Generating Functions, Calculus

Can you hear the shape of a drum? (∗∗∗∗, Dave & Miranda, 4 days)

In other words, do the Dirichlet eigenvalues of the Laplacian on a domain in two dimensions uniquely determine the shape of the domain? This question is a nice way to bring together ideas from physics, differential equations, analysis, functional analysis, and group theory. In this class, we will learn the math needed to be able to ask the question, and then we will answer it. We will start by deriving the wave equation for a vibrating membrane in 2 dimensions, and introduce Green's functions. Then, we will talk about compact operators on infinite-dimensional Hilbert spaces, and discuss when they have an countable sequence of positive eigenvalues. At this point we will be able to understand what the question "Can you hear the shape of a drum" means mathematically. On the last day, we will answer the question, and prove it using group theory.

Prerequisites: For the first 3 days, you should be very comfortable with calculus, and you should also have some previous exposure to analysis - something along the lines of real analysis, measure theory, or point-set topology. For the last days you should know a bit of group theory. In general, you should be prepared for the class to move at a fast pace.

Homework: Maybe

Related to: Fourier analysis.

JR's Classes

(Co)Homology. (∗∗∗∗, JR, 4 days)

The fundamental group is a very useful tool for studying topological spaces. It has limitations, however, stemming from the fact that it is fundamentally a one-dimensional construction. There are higher-dimensional analogs of the fundamental group, called homotopy groups, but they have the drawback of being very hard to compute in practice.

Homology and cohomology have the advantage of being easier to compute than the homotopy groups, while still giving higher-dimensional information about topological spaces. This class will focus largely on simplicial and singular homology and cohomology (the first being defined for triangulated manifolds, the second for all topological spaces), but there will be some discussion of other cohomology theories as well.

Prerequisites: A solid understanding of point-set topology, some group theory, some linear algebra. Familiarity with the fundamental group is helpful, but not required.

Homework: Recommended.

Related to: Algebraic topology.

Random Walks. (∗∗, JR, 2 days)
Imagine a bug sitting somewhere in the middle of a ruler. Every second, there is some probability that the bug crawls one inch to the right, and some probability she crawls one inch to the left. What is the probability she ends her walk on the right side of the ruler, as opposed to the left? How many seconds, on average, will have elapsed when this happens? There is a surprisingly nice technique that can be used to answer these questions.

Prerequisites: None

Homework: Optional

The Gamma Function. (∗∗∗, JR, 3 days)
Most people encounter factorials fairly early on in their study of mathematics. The definition $n! = 1 \cdot 2 \cdot \ldots \cdot n$ makes sense for any positive integer n. Even though it may not be immediately clear from this definition, it can be shown that the "natural" value of 0! is 1. What is less clear, however, is how to interpret an expression like $\left(\frac{1}{2}\right)!$. It turns out that there is a nice function, called Γ, that generalizes the factorial to any real (or even complex) value, and this definition is, in some sense, the only reasonable one.

Prerequisites: Integral calculus

Homework: Optional

The Riemann Zeta Function. (∗∗∗∗, JR, 4 days)
The Zeta function is at the center of one of the most important unsolved problems in mathematics, the Riemann Hypothesis, whose proof is worth one million dollars. In this class, we will explore in depth the relationship between the zeta function and many questions in number theory, including the distribution of the prime numbers. We will prove some results about the distribution of the zeroes of the zeta function, and use the results to prove the prime number theorem.

Prerequisites: Familiarity with the residue theorem from complex analysis. Familiarity with some analytic number theory is useful, but not required.

Homework: Recommended

Julian's Classes

Kakeya Needle Problem. (∗–∗∗, Yvonne & Julian, 2 days)
In 1917, Japanese mathematician S. Kakeya proposed a problem: What is the smallest area through which a needle of length one can be rotated 360 degrees? Clearly a circle of diameter length one – area $\pi/4$ – would do the trick: just pivot the needle about its centre. But we can do better than that: if we take an equilateral triangle of altitude 1 (and hence side length $2/\sqrt{3}$), we can slide the line segment up one side of the triangle, rotate it in the vertex, slide it down the next vertex, and so forth, until the needle is fully rotated. This triangle has area $1/\sqrt{3}$, which is less than $\pi/4$. In fact, a shape called a deltoid, which looks like a triangle whose edges are curved inward, does the trick – and its area is just $\pi/8$ – less than $1/\sqrt{3}$, and just half the area of the circle. Can we do better? In 1928 the mathematician A.S. Besicovich came up with the unexpected answer to Kakeya's problem: no matter how small an area you choose, it's possible to rotate a needle of length one through that a shape with area. More recently, even this solution was improved! Using

fractals, one can construct a *zero area* region in which to rotate the needle. We will discuss both these results in our class.

Day one will be $*$, day two will be $**$.

Prerequisites: High school geometry.

Big numbers! ($*$, Julian, 1 day)

Back by popular demand, a repeat of the class from Mathcamp 2007. We'll be learning a proof of van der Waerden's theorem: if we colour the whole numbers with c colours, there are monochromatic arithmetic progressions of arbitrary (finite) length. (Those who were at the Random Walks in Combinatorics class may recognise this as being from Ramsey Theory.)

Big warning: this class will involve big numbers. I mean, really huge numbers. Astonishingly colossal numbers. In fact, numbers so large that this class comes with a health warning: Do *not* attend this class if you are of a nervous disposition or are scared of large numbers!

Prerequisites: None

How a mathematician reads a newspaper. ($*$, Julian, 1 day)

(A repeat of the class from Mathcamp 2007.)

Maths is full of hypotheses, theorems and logical arguments. What happens when we apply our thinking to a piece of text, say from a newspaper? You will need an open mind, and a willingness to explore a text logically! This session will be heavily based on work by Bandler and Grinder, who developed a model for analysing text based on Chomsky's theory of transformational grammar.

Prerequisites: None.

Fun with Fibonacci. ($*$, Julian, 1 day)

We know about writing numbers in different bases. But what would base Fibonacci look like? And what is the equivalent of 'multiplying by 10' in this new context? We'll explore a fascinating listing of the integers in a Fibonacci-based table which has many curious and surprising properties, first introduced by Conway, and explored by the Queen Mary, University of London's Combinatorics Study Group.

Prerequisites: None

Related to: Wyt Queens.

Parking functions and priority queues. ($*$, Julian, 1 day)

How do you park a lot of cars in a one-way street? And what do you do when you have too many jobs to do? And what's the one got to do with the other? This was one part of my Ph.D. thesis.

Prerequisites: None, except a willingness to take part and have some fun.

Rational trigonometry and universal geometry. ($**$, Julian, 2 days)

Know the intersecting chords theorem? (See below if you don't.) Nice result. At least on the Euclidean plane ($\mathbb{R}^2$). But what would happen if we were to use to a different field, working on say $\mathbb{C}^2$ or $\mathbb{F}_7^2$ (here, $\mathbb{F}_7$ is the integers mod 7)? What could the statement of the theorem mean? And could it still, in some sense, be true?

And is there a way of doing some trigonometry (triangle measuring questions), getting exact answers without needing a calculator or tables? And how could we do trigonometry in $\mathbb{F}_7^2$?!

Come and learn about some recent developments in the fields of trigonometry and (elementary) geometry in these exotic settings!

(The intersecting chords theorem: if AB and CD are two chords of a circle which intersect at P, then AP.PB = CP.PD.)

Prerequisites: Modular arithmetic; complex numbers are useful but not essential.

The motion of the planets. ($*$, Julian, 1 day)

Richard Feynman, the Nobel-prize winning physicist and expositor extraordinaire, gave a lecture in 1964 entitled *The Motion of Planets Around the Sun.* In it, he presented a beautiful geometric proof of Kepler's first law of planetary motion, which states that the path of the planets around the sun is elliptical. This class will aim to reproduce this proof.

The standard calculus proof (as a series of exercises) will be available as a handout.

Prerequisites: None.

Generalised Riemann Integration. ($****$, Julian, 4 days)

We know how to integrate a function such as x^2, but how does one go about defining such an integral precisely? And in what situations does the fundamental theorem of calculus (FTC) break down? (Recall that this theorem states, roughly speaking, that integration and differentiation are inverses of each other. So, for example, $\int x^2\,dx = \frac{1}{3}x^3 + c$ and $\frac{d}{dx}(\frac{1}{3}x^3 + c) = x^2$.) We will learn the formal definitions, and study a generalisation of the standard Riemann integral which provides for the most powerful possible version of the FTC. (For those who have heard of the Lebesgue integral, this one includes Lebesgue as a special case, yet in some sense is even easier to define and understand.)

As an example to chew over, consider the following function:

$$F(x) = \begin{cases} x^2 \sin(1/x^2) & \text{for } x > 0 \\ 0 & \text{when } x = 0. \end{cases}$$

This function can be differentiated on $[0, \infty)$ to get a function f, but f is far too badly behaved at zero to be able to integrate it on $[0, a]$ for any $a > 0$.

Prerequisites: Analysis, in particular ε–δ arguments.

Marisa's Classes

Four Theorems of Erdös. ($**$, Marisa, 3–4 days)

The Hungarian mathematician Paul Erdös is known for being very quirky and very prolific: he published about 1500 papers in his life with 511 collaborators. He spent his adult life wandering from one mathematician's house to the next, "turning coffee into theorems" as he traveled. He solved and posed problems in number theory, combinatorics, probability, set theory, and analysis, and leaves a legacy of great theorems, big and small. In this class, we'll prove one combinatorial result of Erdös each day.

Prerequisites: Basic Graph Theory or equivalent

Related to: Basic Graph Theory, Graphs on Surfaces, Intro Number Theory.

Mark's Classes

The Nine-Point Circle. ($*$, Mark, 1 day)

This class will feature some more or less familiar special points that can be associated with any triangle, quick proofs that those points exist, and some beautiful geometric relationships between them. In particular, we'll look at the nine-point circle (or Feuerbach circle); you can probably guess how many noteworthy points are on that circle.

Prerequisites: None.

Integration by Parts and the Wallis Product. ($**$, Mark, 1 day)

Integration by parts is one of only two truly general techniques for finding antiderivatives (the other is integration by substitution). In this class you'll see (or review) this method, followed by two of its applications: How to extend the factorial function, so that there is actually something like "one-half factorial" (although the terminology in general use is a bit different), and how to derive the famous product formula
$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \ldots$ which was first stated by John Wallis in 1655.

Prerequisites: Basic single-variable calculus.

Homework: None.

Quadratic Reciprocity. ($**$–$***$, Mark, 2 days)

Let p and q be two distinct primes. What, if anything, is the relation between the answers to the following two questions:

(1) "Is q a square modulo p ?"
(2) "Is p a square modulo q ?"

In this class you'll find out; the relation is an important and surprising result which took Gauss a year to prove, and for which he eventually gave six different proofs. If all goes well, you'll get to see all the details of a very nice proof that is in part due to one of Gauss's best students, Eisenstein. And next time someone asks you whether 101 is a square mod 9973, you'll be able to answer a lot more quickly!

Prerequisites: Some intro number theory, specifically Fermat's little theorem.

Homework: Optional.

Primitive Roots. ($**$–$***$, Mark, 1–2 days)

Remember the number "wheels" mod n that you get by repeatedly multiplying by some a that's relatively prime to n? If you can get *all* the numbers mod n that are relatively prime to n on *one* wheel, then the number a for that wheel is called a *primitive root* mod n. In this class we'll study for what moduli n such primitive roots occur, and in particular we'll use a nice counting argument to show that they do occur whenever n is prime.

Prerequisites: Some intro number theory, specifically Euler's theorem, or at least Fermat's little theorem

Homework: Optional.

Determinants, Inverses, and Adjoints. ($**$–$***$, Mark, 3–4 days)

If this class happens, we'll start with a combinatorial definition and a geometric interpretation of the determinant (a number which is associated to any square matrix), and we'll see (and as time permits, prove) basic properties of determinants. Then we'll see how determinants can be used to give a general formula for the inverse of a matrix, and finally we'll use such ideas to prove the striking (and important) Cayley-Hamilton Theorem, which says that any square matrix is a "root" of its own characteristic polynomial. (Don't worry if you don't know what that means - we'll define it.)

Prerequisites: Matrix multiplication. Some experience with polynomials in general and characteristic polynomials in particular would be helpful, but not required.

Homework: Optional.

Partitions and my Erdös number. ($***$, Mark, 3–4 days)

A *partition* of a positive integer n is a way to write n as a sum of one or more positive integers, say in nonincreasing order. For example, the seven partitions of 5 are $5, 4+1, 3+2, 3+1+1, 2+2+1, 2+1+1+1, 1+1+1+1+1$. The number of such partitions is given by the partition function $p(n)$; for example, $p(5) = 7$. An "explicit" formula for $p(n)$ is known, but it's quite complicated and (although tastes differ) not very attractive. On the other hand, there is a beautiful and very efficient recurrence relation due to Euler, which was used (well before the advent of computers) by MacMahon to make a table of $p(n)$ through $p(200) = 3972999029388$. We'll go through a famous combinatorial proof ("Franklin's proof") using pictures to show that the recurrence relation works. Then we'll look at a related theorem called Jacobi's identity; some years ago I went looking for a particular kind of combinatorial proof of this identity, and although I still haven't quite found it, the search did lead to a joint paper with Alex Fink (who is not much older than most campers) and Richard Guy (who is over ninety), and thus brought my Erdös number down to two. As time permits, we'll look at some of the ideas in this paper.

Prerequisites: Geometric series; some experience with generating functions would be helpful.

Homework: Optional, if any.

A Bit of Commutative Algebra and Algebraic Geometry. ($****$, Mark, 4 days)

Classical algebraic geometry is the study of subsets of n-dimensional space (called *varieties*) that are given by sets of polynomial equations. (For $n = 2$ and a single equation, you get an algebraic curve.) Nowadays, algebraic geometry relies heavily on commutative algebra. We'll introduce some basic algebraic ideas, such as prime and maximal ideals and quotient rings, and then see how some geometric ideas, such as dimension, can be defined and/or studied using the language of algebra.

Prerequisites: Some knowledge of rings, especially polynomial rings. Previous experience with quotient rings or factor groups would be helpful.

Homework: Optional.

Fourier Series. ($***$, Mike, 4 days)

Long have you dreamt of writing the function $f(x) = x$ as a complicated infinite series involving sine waves. Does your dream come true? Yes!

$$x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx), \qquad -\pi < x < \pi.$$

In fact, a wide range of functions can be written as sums of sine waves. In this way we can exploit symmetry that such functions don't obviously have, without losing any information about them.

Fourier series were originally developed to solve PDE (partial differential equations), and we'll touch on this, as well as other applications, including a surprising proof that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}.$$

Prerequisites: Limits, continuity, infinite sums, derivatives, integrals, complex arithmetic. You should be familiar with what an orthonormal basis is.

Homework: Recommended.

Related to: Math of Music, Real Analysis, Measure Theory, Analytic Number Theory, Fractal Dimension

Fractal Dimension. ($**$, Mike, 2 days)

For living in and for demolishing the camper team at ultimate frisbee, dimension 3 is fine, but why not dimension 0.8675309? Fractal dimension is a way to describe how large a geometric set is, be it a plain old shape, something beautifully intricate and self-symmetric, or something hopelessly nasty. We'll try to get an idea of what a more general definition of dimension should mean, how to compute it for examples like the Cantor Set, the Sierpinski Gasket, and the Koch Snowflake, and how to find a set of any specified dimension.

Prerequisites: Limits, logarithms.

Homework: Optional.

Related to: Measure Theory, Real Analysis, Dynamical Systems.

Sexy Complex Analysis. ($***$–$****$, Mike & Nina, 4 days)

This title is totally redundant because Complex Analysis is the sexiest thing in the universe.

The following is a dramatization. The people involved are fictional. Any resemblance to persons and their viewpoints, real or imaginary, is an entire coincidence.

Mina: Isn't it the sexiest fact in the world that you can map *any* region that isn't the whole complex plane to the unit disk in a way that all angles are preserved?

Nike: That is so not sexy. The sexiest fact is that a differentiable function on a compact region always assumes its maximum modulus on the boundary of that region.

Mina: That is pretty cool, but it's so much sexier that any non–constant function differentiable on the whole complex plane takes on all complex values, except possibly one of them.

Nike: You want sexy? A differentiable function is completely determined by its values at a collection of points clustering at some point of differentiability.

Mina and Nike: Wow, there's just so much stuff that's sexy about functions of a single complex variable, we just can't wait to learn more about them.

Prerequisites: Limits, sequences, series, integration, differentiation, complex numbers and complex arithmetic (handout and problems available if you haven't seen this before).

Homework: Required.

Related to: All math that is sexy, including Number Theory, Real Analysis, and Hyperbolic Geometry.

Mira's Classes

Information Theory and the Redundancy of English. (∗∗-∗∗∗, Mira, 4 days)

NWSFLSH: NGLSH S RDNDNT!! (DN'T TLL YR NGLSH TCHR SD THT)

The redundancy of English (or any other language) is what allows you to decipher the above sentence. It's also what allows you to decipher bad handwriting or to have a conversation in a crowded room. The redundancy is a kind of error-correcting code: even if you miss part of what was said, you can recover the rest.

How redundant is English? There are two ways to interpret this question:

- How much information is conveyed by a single letter of English text, relative to how much could theoreticaly be conveyed? (But what *is* information? How do you measure it?)
- How much can we compress English text? If we encode it using a really clever encoding scheme, can we reduce the length of the message by a factor of 2? 10? 100? (But how will we ever know if our encoding is the cleverest possible one?)

Fortunately, the two interpretations are related. In this class, we will first derive a mathematical definition of information, based on our intuitive notions of what this word *should* mean. Then we'll prove the Noiseless Coding Theorem: the degree to which a piece of text (or any other data stream) can be compressed is governed by the actual amount of information that it contains. We'll also talk about Huffman codes: the optimal way of compressing data if you know enough about its source. (That's a big "if", but it's still a very cool method.) Finally, we'll answer our original question – how redundant is English? – in the way that Claude Shannon originally answered it: by playing a game called Shannon's Hangman and using it as a way of communicating with our imaginary identical clones!

The class is 4 days long, but you can skip some of the days and still come to the others. Here's how it works:

Day 1 ∗∗: *Introduction and definition of information.* Required for the rest of the class, unless you've seen some information theory before.

Days 2, 3 ∗∗∗: *Noiseless coding and Huffman codes.* The mathematical heart of the class, where we'll prove the Noiseless Coding Theorem.

Day 4 ∗∗: *Shannon's Hangman and the redundancy of English.* You can come to this class even if you don't come on Days 2 and 3 – you just need the material from Day 1.

Prerequisites: None.

Homework: Recommended.

Four Mathematical Marriage Problems. (Mira, ∗∗, 3-4 days)

> You may think that marriage is outside the scope of applied mathematics. That is true. Once you are married, mathematics is of no possible use. But there is a superficial model for a simpler problem – getting married in the first place...
>
> – Gilbert Strang, *Introduction to Applied Mathematics*

Welcome to Mira's Mathematical Counseling for Stable Partnerships!

Want to know how many people you should date to maximize your chance of eventually finding true love? Ever wonder who has the better deal in 19th century novels: the men, because they get to propose, or the women, because they get to choose? Believe it or not, there are actual mathematical answers to these questions! Sure, the model of marriage they assume is somewhat simplified, but it's not entirely divorced from reality. And the math turns out to model other situations too, with practical applications that extend far beyond the problem of choosing a spouse.

Nor does the mathematics end once you get married, whatever Professor Strang might say. Here is a passage from the Mishnah – the 2nd century codex of Jewish law:

> "A man has three wives; he dies owing one of them 100 [silver pieces], one of them 200, and one of them 300. If his total estate is 100, they should split it equally. If the estate is 200, then the first wife gets 50 and the other two get 75 each. If the estate is 300, then the first wife gets 50, the second one 100, and the third one 150. Similarly, any joint investment with three unequal initial contributions should be divided up in the same way."

For 1800 years, this passage has baffled scholars: what could possibly be the logic behind the Mishnah's distribution of the estate? Then, in 1985, a pair of mathematical economists produced a beautifully simple explanation based on ideas from game theory. They showed that for any number of creditors and for any estate size, there is a unique distribution that satisfies certain criteria – and it turns out to be exactly the distribution proposed in the Mishnah. See if you can figure out the logic for yourself, or come to class and find out.

Prerequisites: None.

Homework: Optional.

Miranda's Classes

Can you hear the shape of a drum? (∗∗∗∗, Miranda & Dave, 4 days)

(See Dave's classes.)

Asymptotic Methods. (∗∗∗, Miranda, 4 days)

What is $\lim_{x\to\infty} \int_x^\infty e^{-t^4} dt$, as a function of x? If $y_\epsilon(t)$ is the solution to $\epsilon y'' + y' + 1 = 0$, and $y_0(t)$ is the solution to $y' + 1 = 0$, how does $\lim_{\epsilon\to 0} y_\epsilon$ compare to y_0? In this class we will look at how to approximate things like integrals, and solutions to differential equations, when there is a small (or large) parameter involved. We will learn about perturbation series, and use them to express solutions as a sum of functions which diverges! We will look at boundary-layer problems, and if we have time, look at multi-scale analysis.

Prerequisites: Calculus, and the ability to find simple limits.

Homework: Maybe.

Chase and Escape Problems. (∗∗-∗∗∗, Miranda, 2-4 days)

If a rabbit is trying to run away from a fox, and there is a tree between them, in what direction should it run so that the tree is always between them? And is this always possible? How does a pirate ship catch a ship full of gold which is travelling in a straight line, if all it knows is the initial position of the ship but not its direction? Problems like these, where one thing is chasing another, which is in turn trying to avoid being caught, are called 'pursuit' problems, and studying them is a nice application of calculus and the differential geometry of curves. This class would look at a few problems and ways to solve them - and maybe a few unsolved ones too!

Prerequisites: Should be comfortable with calculus.

Homework: None.

More Math of Music. ($*$, Miranda, 1 day)

If we don't have enough time in Week 4 to talk about musical paradoxes, and how our brain perceives music, then this is what we'll do in Week 5. If we do cover this in Week 4, then we can spend another day listening to demonstrations of musical experiments, such as critical bands, just noticeable difference, and different kinds of masking.

Prerequisites: None: you will be able to follow most of this class, even if you didn't come to Math of Music in Week 4.

Nina's Classes

Seifert Surfaces. ($*$, Nina & Susan, 1 day)

Given a knot, we can always construct a surface whose boundary is the knot; that is, when we lay a string along the edge of the surface, we recover the knot. A straight-forward algorithm, using checker-coloring, gives us this surface for every knot. Come see how it's done!

Prerequisites: None.

Related to: Knot Theory.

Sexy Complex Analysis. ($***$–$****$, Nina & Mike, 4 days)

(See Mike's classes.)

Quotients in *insert category name here*. ($***$, Nina, 2–4 days)

We see quotients in many categories: groups, topological spaces, vector spaces, rings, sheaves. There is a way to describe these quotients in these different categories in a unified way. We call this the universal property of a quotient. (Similar descriptions exist for other common constructions like various products.) Let's see what these words "universal property" mean, and see useful examples of quotients in various categories.

Prerequisites: Group Theory, Topology... If you've seen universal properties before, don't come.

Homework: Some.

Related to: Dan's Limits and Colimits, Category Theory, Functors.

Noah's Classes

Basis Free at Last! ($***$, Noah, 3 days)

The usual definitions of, say, trace and determinant require choosing a basis. However, trace and determinant are independent of that choice of basis. In this class we'll discuss how to give definitions that don't depend on a choice of basis.

Homework: Recommended.

Prerequisites: A little linear algebra.

The Outer Automorphism of S_6. ($**$, Noah, 1 day)
The symmetric group on 6 letters is the only symmetric group which has an outer automorphism. I'll explain where this automorphism comes from!

Prerequisites: Two weeks of basic group theory.

The Riemann Hypothesis for Polynomials. ($***$, Noah, 1 day)
I'll prove the Riemann Hypothesis for polynomials, thus getting a million polynomial dollars. This is one of my favorite proofs ever.

Prerequisites: The Mobius inversion formula, Euler factorization of the zeta function. These were both covered in Analytic Number Theory.

Related to: Analytic Number Theory.

Topological Quantum Computation. ($***$, Noah, 1 day)
As Scott Aaronson and Alice have discussed, you can use quantum mechanics to get a new model of computing that seems to be stronger than classical computing. What if instead of quantum mechanics you use a more exotic quantum system? For example, take a very thin sheet, make it really really cold, and put it in a strong magnetic field, and you end up with something called a Topological Quantum Field Theory, or TQFT. Is a TQFT computer better or worse than an ordinary quantum computer? Why would you want to build a TQFT computer?

Prerequisites: None.

Related to: Quantum Computation, Planar Algebras, Knot Theory.

TBA. ($****$, Noah, 1 day)
I can't explain what this class is on until after the puzzle hunt.

Prerequisites: TBA.

Related to: TBA.

Unique Factorization and Ideals. ($***$, Noah, 1–2 days)
In $\mathbb{Z}[\sqrt{5}]$ we see that $(1+\sqrt{5})(1-\sqrt{5}) = 2 \cdot 3$. Thus elements of $\mathbb{Z}[\sqrt{5}]$ do not have unique factorization. However, this can be fixed by introducing "ideal numbers," which don't really exist, but do allow you to recover unique factorization!

Prerequisites: Some number theory.

Related to: $\mathbb{Q}(\sqrt{d})$ in Space.

Planar Algebras cont'ed. ($***$–$****$, Noah, 2–3 days)

Prerequisites: Planar algebras.

Dense Linear Orderings. (Susan, $***$, 3 days)

Take the field of real numbers. Now forget about multiplication and addition. What's left over? The elements no longer relate to each other through their operations, but they can still be put in order. In this class we will be studying sets like the rationals, with no structure on them except their order. We'll also learn how to differentiate between different DLOs, find out exactly how many countable DLOs there are, and learn how to play the transfinite back-and-forth game.

Prerequisites: Set theory.

Homework: Optional.

Intro to Models. (Susan, $**$, 3 days)

Go through your notes and count the number of axiom systems you've used in the past four weeks. Ever wonder why or how axioms work? Learn about the structures behind the axioms. Also find out how to define an order on a ring, and why the rational numbers know where π is.

Prerequisites: Some math logic.

Seifert Surfaces. ($**$, Susan & Nina, 1–4 days)

(See Nina's classes.)

Yvonne's Classes

The Gauss-Bonnet Theorem. ($**$, Yvonne, 1-2 days)

Suppose that I give you a rubber sphere that you can poke, squish, and stretch as much as you like. When you give it back to me, I compute the following integral on it:

$$\int_{\text{sphere}} (\text{curvature})\, \mathrm{d}(\text{Area})$$

... and I obtain 4π, which is exactly 2π times the Euler characteristic of the sphere. Replace with sphere with a rubber donut, and I obtain 0, which is again 2π times the Euler characteristic. It seems that we can detect the topology of the surface simply by looking at the local geometry. (If you have enough ants on your beach ball, they can tell you what shape it is.) This mysterious connection between geometry and topology is furnished by an elegant and elementary proof, which we will see in this class. Time permitting, we will cover piecewise linear analogs, which allow us to define curvature by tiling a surface with polygons.

Prerequisites: You should be familiar with the equation $V - E + F = 2$ on the sphere. No calculus–we'll be able to do everything with extremely friendly sums.

Homework: none

Related to: Classification of Surfaces, Graphs on Surfaces

Arts and Crafts in Hyperbolic, Euclidean, and Spherical Space. (∗, Yvonne, 2 days)

Do you own a model of hyperbolic space? Why not? Make and bring home your very own piece of hyperbolic space. If you think you're familiar with 3-space, come to this workshop. We'll construct wacky dodecahedral 3-spaces that don't initially look at all like the one we live in, but were in fact recently proposed candidates for the shape of our universe.

All (Euclidean 3-space embeddable) construction materials will be provided.

Prerequisites: none

Kakeya Needle Problem. (∗–∗∗, Yvonne, Julian, 2 days)

(See Julian's classes.)

Visitors' Classes

The Jones Polynomial. (∗∗-∗∗∗, Ari Nieh, 1 day)

The Jones Polynomial, concocted in the early 80's, is a powerful invariant for knots and links. Unlike the Alexander Polynomial, it can distinguish between the right- and left-handed trefoil knots. Jones' discovery led to a whole zoo of new "quantum" knot invariants, more of which are still being researched today! Although Jones first came upon it via some fairly high-powered machinery, we will follow Kauffman's approach to create the polynomial from scratch with extraordinarily reasonable rules. In the end, we'll see that the invariant can essentially be defined by a single equation called a "skein relation", and see how this relates to other knot invariants.

Prerequisites: None.

Related to: Knot theory, Planar algebras.

The Real Projective Plane. (∗∗∗, Brenda Fine, 4 days)

Welcome to the Real Projective plane - a place where points and lines are interchangeable, where ellipses and hyperbolas trade places at will, and where infinity can be reigned in for a closer look. The real projective plane - which we can model by adding a single line "at infinity" to the standard Euclidean plane - provides us with some powerful tools that provide simple and elegant proofs of many familiar theorems in Euclidean geometry.

Prerequisites: Basic linear algebra - you should know what matrices and vectors are.

Homework: Optional

Related to: Moon's colloquium.

Inversive Geometry. (∗∗, Brenda Fine, 2 days)

Here's a problem that's simple to state, but difficult to attack using the standard tools of Euclidean geometry: given three circles, two of which are tangent to one another, construct all of the circles that are tangent to all three. In this class, we'll look at a simple but powerful transformation called inversion, which effectively amounts to "turning a circle inside out" by sending the points inside the circle outside, and vice versa. Using this method, we can turn the above question about three circles into an easier one involving a circle and two lines. Over three classes, we'll look at a handful of surprising and elegant results in Euclidean geometry that can be proven using this tool.

Prerequisites: None.

Calculus Without Calculus. (∗∗, Brenda, 2 days)

If you've ever taken a calculus class, you've almost certainly seen certain types of problems. Without a doubt, you've learned how to find the equations of tangents to curves. In all likelihood, you've learned how to maximize an area with a given shape and perimeter, and minimize the perimeter of a region of given shape and area. You've probably also seen the ol' "swim-and-run" problem of finding the route that minimizes the amount of time it takes to swim to the shore, and then run to a certain place on land. As it turns out, all of these problems - and more - can be solved without evaluating a single limit or derivative. In this brief course, we'll exploit the geometric properties of diagrams, and we'll explore some powerful inequalities that let us solve optimization problems swiftly and elegantly. Come to Calculus Without Calculus, and learn the math your calculus teacher doesn't want you to know. Previous knowledge of calculus isn't necessary, and in fact, those who have taken a calculus class before may find themselves distressed to learn just how much calculus they've used unnecessarily in the past.

Bidding Games. (∗, Sam Payne, 1 day)

A bidding game starts with a familiar two player game such as tic-tac-toe, connect four, or chess. Play as usual except, instead of alternating moves, you bid for the right to move. For instance, we could play bidding chess starting with one hundred chips each. If I bid twelve for the first move and you bid fifteen, then you give me fifteen chips and make a move on the chess board. Now I have 115 chips and you have 85, and we bid for the next move . . . Even tic-tac-toe is surprisingly subtle as a bidding game. Don't believe me? Let's play. This ain't yo grandma's tic-tac-toe.

(Note: Bidding tic-tac-toe is now available as a facebook app. Visit `http://apps.facebook.com/biddingttt` and play online!)

Prerequisites: Yo mama filled yo prereqs.

Related to: Combinatorial game theory.

Homework: Last night.

Bidding Hex. (∗∗, Sam Payne, 1 day)

Hex is played on a board tiled by hexagons. One player tries to make a connected path of black hexagons from the top left edge to the bottom right, while the other player tries to make a connected path of white hexagons from top right to bottom left. The game is popular among mathematicians and computer scientists, but writing artificial intelligence to play Hex well is a notoriously difficult problem (moderately skilled human players still beat the best AI consistently).

Surprisingly, Bidding Hex is a much simpler game than regular, alternating-move Hex. Don't believe me? Let's play. My computer will beat you every time.

Prerequisites: None.

Related to: Combinatorial Game Theory, Bidding Games.

Homework: Bidding games practicum. Absolutely mandatory. (In other words, attend Sam's bidding game casino on Tuesday or Wednesday night.)

Brouwer's Fixed Point Theorem and the Game of Hex. ($***$-$****$, Sam Payne, 1 day)

Hex is a board game with the simplest of rules—two players compete to form a connected path across the board—-but the play is nearly as intricate as chess or Go. John Nash came up with a clever proof that the first player has a winning strategy in 1948, but still no one knows what it is. In this class, I will introduce the game of Hex and show you why, unlike tic-tac-toe, it can never end in a draw. Then I will use this fact to give a simple proof of Brouwer's Fixed Point Theorem: every continuous map f from a disc to itself has a fixed point, which is a point x in the disc such that $f(x) = x$. In other words, you can't move every point in a disc without ripping it.

Prerequisites: Understand the definition of a continuous map: "for every ϵ there is a δ such that..."

Homework: Construct a map from the disc to itself with no fixed points.

Related to: Combinatorial game theory, topology.

Graphs and Rings and Other Such Things. ($**$, Sarah Fletcher, 1–2 days)

At my REU this summer I have been studying the zero divisor graphs of rings. In particular, I have been attempting to prove that that every finite local ring shares its zero divisor graph with another finite local ring of characteristic p. On one level, the details in trying to understand what is going on with these creatures are quite technical and would require a much higher star rating and/or some advanced prereqs. My goal for this class is not for you to understand these technical details. Instead, my goal is to give you a feel for the cool stuff I have been playing with all summer!

Prerequisites: Familiarity with polynomials.

Related to: Grap theory, abstract algebra.

Visitor Bios

Ari Nieh. Ari has been a Mathcamp student, JC, or mentor every summer since 1996. He received his PhD from the University of California at Berkeley in 2007, studying quantum topology. This fall, he is moving to Boston to begin work on a graduate performance diploma at Longy School of Music. He is also the creator of the Sarong Theorem Archive (google it!)

Brenda Fine. Once upon a time, Brenda Fine studied tropical geometry at UBC. These days, when she's not mucking around in a pottery studio or playing Celtic fiddle, she can be found teaching math at BCIT. She gets hugely excited about the real projective plane.

Sam Payne. Sam Payne was a four-time Mathcamp mentor (02, 03, 05, 06), teaching classes on topology, polytopes, algebraic curves, Young tableaux, and the awesomeness of Archimedes. He is currently a postdoc at Stanford, specializing in algebraic geometry by day and bidding games by night. Sam is a cross between a rock star and the incredible hulk.

Sarah Fletcher. Sarah is a math major in the class of 2009 at Harvey Mudd and was a camper in 2004 and a JC in 2006 and 2007. This summer she succumbed to pressure from her professors to do an REU (Research Experience for Undergraduates) instead of applying to be a JC again (something about it being good practice for thesis and grad schools liking it and . . .). She also spent last fall in Hungary through the Budapest Semesters in Mathematics program and enjoyed it so much that she is now strongly considering going abroad for grad school.